The Ideas of Evariste Galois:

Recovering Motivation in Abstract Algebra Through the Exploration of Original Sources

Pew Research Grant Proposal - Fall 2000

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I. Description of the projects and its major goals.

Dr. Laszlo Fuchs, the Phillips Distinguished Professor of Mathematics at Tulane University and my dissertation advisor, taught me that teachers of mathematics “must concentrate not on the formalisms but on the ideas behind them, putting more emphasis on motivation and practical uses and stimulating creative thinking.” It is from this point of view that I approach the research project described below. Mathematics is a process and that process is often hidden from even our best students. Mathematics professors can mislead students by presenting the discipline as abstract concepts that appear not only void of practical use but also disconnected from the ideas that motivated them. John Stillwell says in his text Elements of Algebra that "it seems to be one of the laws of mathematical history that if a concept *can* be detached from its origins, it will be."

This is certainly the case in the area of mathematics commonly referred to as abstract algebra. Joseph Gallian in his book Contemporary Abstract Algebra states, “The goal of abstract algebra is to discover truths about algebraic systems (that is, sets with one or more binary operations) that are independent of the specific nature of the operations.” While this modern abstract approach, mainly due to its generality, provides a unifying element to the study of mathematics, it conceals from the student many of the great ideas generated by significant problems in the history of the discipline. B. Melvin

Kiernan asserts that "without a clear historical perspective it is difficult to see or even imagine the connection between the [abstract] algebra of the present day and the computational problems from which it arose." Therefore, it seems reasonable to suggest that by incorporating historical perspective and original ideas into the study of abstract algebra, mathematics professors could enhance the learning process for the student as well as improve their own pedagogy.

This summer, I propose to study the mathematical ideas of Evariste Galois. Galois (1811-1832) was born near Paris and lived his short life during a very turbulent time in the history of France. By the age of sixteen, Galois knew that he was a very gifted mathematician. However, in his lifetime, his mathematical abilities never gained the recognition they deserved. Twice he was denied admission to the Ecole Polytechnique. One paper sent to the prestigious Academie des Sciences in Paris was apparently misplaced by Cauchy and a second paper was lost when Fourier, the secretary of the Academy, died. His third submission resulted in a return of the paper by Poisson with a request for proofs. Obviously disillusioned, Galois turned his attention to the political issues of France. In 1831, he spent several months in jail as a political prisoner of King Louis Philippe. Shortly after his release, Galois accepted a challenge to a duel. On the night before, he wrote a letter to his friend and schoolmate Chevalier which contained notes on his mathematical discoveries. Galois was shot during the duel and died the next day. He was twenty years old. In 1846, the French mathematician Liouville edited several memoirs and manuscripts of Galois and published them along with the letter to Chevalier in the *Journal de Mathematiques*. This publication marks the beginning of the dissemination of Galois' ideas.

I want to focus my research on two famous memoirs of Galois: *Memoirs on the Conditions for Solvability of Equations by Radicals* and *Primitive Equations that are Solvable* *by Radicals*. The publication of these papers in 1846 marks the official beginning of Galois Theory, a subject that has exerted an enormous influence on the development of abstract algebra. Galois was the first published mathematician to introduce the concept and the term "group" in its technical, mathematical sense. Today, the study of groups is foundational to any introductory course in abstract algebra. Furthermore, Galois identified a specific property of groups, now known as solvability, that enabled him to translate his original problem from the theory of equations into an equivalent problem within the newly established theory of groups. The solvability of a group is determined by examining the subgroup structure of the group. The 20th century algebraist I. N. Herstein said, "It is a tribute to the genius of Galois that he recognized that those subgroups for which the left and right cosets coincide are distinguished ones. Very often in mathematics the crucial problem is to recognize and to discover what are the relevant concepts; once this is accomplished the job may be more than half done." Today, these special subgroups are known as "normal" subgroups and they are studied by every student of abstract algebra. The concepts of group, normal subgroup, and solvability are vital to an understanding of abstract algebra. Therefore, I want to explore the historical context and the original mathematical ideas that brought these significant concepts into existence.

One might ask why a mathematician would need to do research in order to recover the original ideas of a fellow mathematician. The answer is simple: Galois expressed his thoughts in the mathematical language of his time. It is indeed a challenge for a contemporary mathematician to read the original works from even a century ago. The modern presentation of Galois Theory hides the context and notions that I am so eager to examine. Furthermore, an understanding of the memoirs of Galois will require an awareness of the knowledge base from which Galois worked. Contributions of other mathematicians (e.g. Lagrange, Gauss, Cauchy, Abel) who influenced Galois will need to be reviewed as well.

Upon completion of my study of these memoirs, I want to be able to answer this question: How can the ideas of Galois as published in his memoirs, along with a thorough discussion of the historical perspective of his work, enhance the teaching of undergraduate abstract algebra? I wish to write an article, which I plan to submit for publication in a peer reviewed journal, elucidating the original ideas of Galois and indicating how a professor of mathematics can use these ideas within their appropriate context to motivate the study of undergraduate abstract algebra. I see this project as the first installment of a much larger endeavor. Eventually, I would like to identify a collection of great ideas throughout the history of abstract algebra and explore their pedagogical value within the contemporary undergraduate mathematics curriculum.

The famous Norwegian mathematician Neils H. Abel said, "It appears to me that if one wants to make progress in mathematics, one should study the masters and not the pupils." This research project is all about studying one of the masters--Evariste Galois. The dissemination of this research will facilitate the study of one of the masters for a wide range of students and professors of mathematics.

II. Review of Scholarly Literature

Although much has been written about the brief life of Evariste Galois and about the branch of abstract algebra known as Galois Theory, to my knowledge very little has been written about how one might use the original works of Galois as motivation for the study of contemporary abstract algebra. I was introduced to the concept of teaching with original sources by Laubenbacher, Pengelley, and Siddoway in their paper *Recovering Motivation in Mathematics: Teaching with Original Sources* [1994]. While these authors claim that there is a vast supply of original sources available to the mathematics professor, I would argue that most professors fail to possess the understanding of historical perspective and succession of ideas needed to incorporate these works into the undergraduate curriculum. While excerpts of Galois' memoirs do appear in several mathematics source books, these excerpts may not accomplish the goal of moving either the instructor or the student beyond the obvious fact that Galois was the originator of significant ideas. There is large gulf between reading selected excerpts of original sources with commentary and understanding what the author was actually thinking when he wrote his results. Thus, I believe that the research project I have outlined above is novel. It is an attempt to integrate the scholarship that exists on the historical development of Galois Theory with the pedagogical scholarship that calls for motivation of mathematical ideas through the use of history and original sources.

The historical evolution of Galois theory from its beginnings up to its formulation by Emil Artin in the late 1930's can be found in B. Melvin Keirnan's *The Development of Galois Theory from Lagrange to Artin* [1971] and B. L. van der Waerden's *Die Galois-Theorie von Heinrich Weber bis Emil Artin* [1972]. In particular, Keirnan discusses the origins of Galois' ideas in the works of Lagrange, Ruffini, Gauss, Abel, and Cauchy. Two books, <u>Galois Theory</u> [1984] by H. M. Edwards and <u>Galois' Theory of Algebraic Equations</u> [1988] by J. P. Tignol examine Galois Theory from an early 19th century perspective. The Edwards book includes an English translation of *Memoirs on the Conditions for Solvability of Equations by Radicals.* An unpublished dissertation on *Primitive Equations that are Solvable by Radicals* also exists. The influence of Galois' ideas is addressed in many books on abstract algebra including: <u>Elements of Algebra</u> [1994] and <u>Mathematics and Its History</u> [1989] by John Stillwell, <u>Galois Theory</u> [1990] by Joseph

Rotman, and A History of Algebra [1985] by B. L. van der Waerden. A recent comprehensive biography Evariste Galois (1811-1832) [1996] has been written by Laura Toti Rigatelli.

There is growing amount of scholarship on the use of the history of mathematics and original sources in the teaching of the discipline. During the 1990's, the National Science Foundation funded a multi-year Institute on the History of Mathematics and Its Use in Teaching (IHMT). In addition to the article mentioned above, Laubenbacher and Pengelley have written several others on the use of original sources including *Mathematical Masterpieces: Teaching with Original Sources* [1996], which describes a senior-level mathematics course developed at New Mexico State University. The Mathematical Association of America (MAA) has published two recent texts: Vita Mathematica: Historical Research and Integration with Teaching [1996] and Learn From the Masters [1995] which contain significant historical ideas and insights and thus enable professors to incorporate these ideas into the undergraduate curriculum. In addition to these articles and texts, several source books have been recently published including Classics of Mathematics [1995] by Calinger and The History of Mathematics: A Reader [1987] by Fauvel and Gray. These books include excerpts from original works, with valuable commentary. In a related vein, Israel Kleiner of York University has written an article A Historically Focused Course in Abstract Algebra [1998] that describes a course for secondary teachers of mathematics. Kleiner uses history to motivate the study of abstract algebra through a problem-based discovery approach. In the course, Kleiner uses only secondary sources to accomplish his goals.

III. Time Frame for Completion and Dissemination of the Project

Spring 2001

Identify sources that need to be read or reviewed. Plan trip and itinerary.

June-July 2001

Travel to Toronto. I anticipate spending one week visiting the Institute for the History and Philosophy of Science and Technology at the University of Toronto. The Institute has an impressive library containing all standard reference works in the history of mathematics including all issues of *Historia Mathematica*.

Travel to Louisville. I plan to visit the Bullitt Rare Books Collection at the University of Louisville. The Collection includes an original version of Liouville's 1846 publication of Galois' work with four page introduction.

August 2001

Write draft of article.

Fall 2001

Finish article and submit for review and publication.

Spring 2001

Report to Pew Selection Committee.

IV. Budget

Travel (Toronto, Louisville)	$575
Lodging (Toronto, Louisville)	750
Food	350
Misc expenses	200
Salary (equivalent of one summer course pay)	1625
Total	**$3500**

V. Essay on the integration of faith and discipline

The integration of faith and learning naturally leads to an examination of the historical and philosophical foundations of the discipline. The Christian scholar should engage in conversations concerning intellectual events and significance of ideas within the discipline. The Christian teacher should probe *why* certain disciplinary areas of knowledge are selected for inclusion in the curriculum and *how* these specific areas are presented to students. The research project outlined above is an investigation of the historical foundations of one area of mathematics, namely abstract algebra. It focuses on the ideas of one of the subject's primary and prominent contributors - Evariste Galois. Its goal is simply the recovery of perspective and motivation in the undergraduate presentation of the subject.

Mathematics is a human endeavor. The ability to *do* mathematics is a gift from God. I *do* mathematics because I enjoy it, it is significant, and because I realize the talent I have received from my Creator. Furthermore, because of the views professed below, I believe that every member of society should possess some degree of mathematical literacy. Therefore, I consider it my responsibility, my *vocatio*, not only to *do* mathematics but also to *teach* mathematics. It is a privilege to share my discipline and to make it more enjoyable and accessible to students and the greater community.

Albert Einstein marveled that, "the only incomprehensible thing about the universe is that it is comprehensible." Remarkably, God has privileged humanity with the capability to explore His creation through the use of reason and rational thought and to draw necessary conclusions from His design. Mathematics is the intellectual activity that often best anticipates and expresses the outcomes of this exploration; it is, perhaps, the principal instrument through which understanding is achieved.

The matter of the "unreasonable effectiveness" of mathematics has been debated for centuries. Since the time of the ancient Greeks, philosophers, mathematicians, scientists, and theologians have wrestled with the obvious fact that mathematics describes nature so satisfactorily. Some would argue that this is no more than a cultural phenomenon; that is, humans have chosen to view the world through the lens of mathematics. Therefore, it is no mystery why the fundamental laws of physics are mathematical--we have simply defined as fundamental those laws that are mathematical.

I disagree with this viewpoint. The mathematical physicist Paul Davies in his book The Mind of God [1992] asserts that the cultural hypothesis can not explain the following challenges: 1) why is it that so much of the mathematics that best describes physical theory was first communicated as abstract mathematics by pure mathematicians who knew of no need for its application? (e.g. non-Euclidean geometry), 2) how is it that the mathematically based laws of physics that disclose reality are truly superb? (e.g. general theory of relativity), and finally 3) how can we justify the extraordinary mathematical ability of the human brain and the existence of mathematical prodigies and geniuses?

While Davies' arguments may not be conclusive, I would state that faith together with the "unreasonable effectiveness" of mathematics points us to a rational Creator, Who, because of the "unreasonably effective initial conditions" of His created universe, has placed us in a rational, intelligible world. Furthermore, as creatures who reflect His image, we have been given the ability to think deeply about His creation. Thus, the question of the "unreasonable effectiveness" of mathematics finds its answer in God.

Evariste Galois also introduced the idea of finite field, also called the Galois field. Galois and Politics. Galois got involved in politics and this led to his expulsion from Ecole Normale. In 1831, Galois quit school and joined the artillery unit of National Guard. He divided his time between mathematics and politics. In the same year, Galois was heading a protest, wearing a uniform of the disabled artillery. He was heavily armed with pistols, a dagger, and a rifle.Â Evariste Galois returned to his work on mathematics after being expelled from Ecole Nomale. He tried to start private classes in advanced algebra, but he was not successful. He still continued to participate in political activities. On May 30, 1832, Galois was shot during a duel and he died the next day. The Ideas of Evariste Galois:Recovering Motivation in Abstract Algebra Through the Exploration of Original Sources. Galois and his groups (and references therein). Original Works of Evariste Galois, on math.SE (and references). The (not so) simple idea of Galois Theory, on scribd. Galois theory after galois. Galois Theory for Beginners. Specificaly the last reference (7) makes explicit the previous analysis and derives the central result of Galois using basic abstract algebra: The aim of this paper is to prove the unsolvability by radicals of the quintic (in fact of the general nth degree eq Concrete algebra versus abstract algebra. Galois theory can be given as a self-contained course in abstract algebra: eld extensions and their automorphisms (symmetries), group theory. I hope to be able to shake free of this tradition, which is distinctly old-fashioned. My aim in this section has been to show that much of the time, Galois theory is closely related to concrete calculations. Beyond that, Galois theory is an important component of many other areas of math beyond eld theory, in-cluding topology, number theory, algebraic geometry, representation theory, dierential equations, and muc