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The Singaporean Mathematics Curriculum: Connections to TIMSS

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Singaporean students' top-level performance in the TIMSS studies in mathematics has strong connections to the mathematics curriculum in the country. While mathematics curricula tend to be somewhat similar in countries all over the world, there are some subtle differences in the Singaporean mathematics curriculum, which are worth examining. In this paper, I describe some of the important features of the intended, attained and implemented aspects of the mathematics curriculum in Singapore and how they connect to the TIMSS studies. I also briefly comment on the strengths and weaknesses of the curriculum.

Mathematics curricula all over the world have certain features in common. The Singaporean mathematics curriculum document is probably no different from what exists in many other countries. Yet, there are subtle differences in the Singaporean mathematics curriculum that makes it worthwhile to examine. In this paper, I give a background of educational system in Singapore and I briefly comment on the essential components of the Singapore Mathematics Curriculum Framework (SMCF). The focus is not on the details of the curriculum content but on the more general features. I also comment on the connections of the SMCF to Singaporean students' performance in the TIMSS studies and what are some of the perceived strengths and weaknesses of the SMCF. A little background seems fitting.

Background

Singapore is a small island state off the tip of the Malaysian peninsula which became independent in 1965. The country has progressed from very modest origins (GDP of \$300 in 1970s) to the status of a developed nation (GDP of \$25000). The only resource the country can bank on is its multicultural human resource, now close to 4.2 millions. The country has a total of 355 schools (175 primary schools and 162 secondary schools with 16 Junior Colleges and two centralized institutes) for a school population of about 530 000. Public expenditure on education is about \$4 billion which is about 4% of GDP.

As a former British colony, Singapore has a system of education modelled largely on the traditional British system. After independence, Singapore continued her collaboration with the University of Cambridge Local Examinations Syndicate to develop a curriculum more suited to her local needs. Referring to the secondary level, Lee and Fan (2002) claimed that the first Singapore local syllabus in mathematics was drafted in 1957 and published in 1959; called syllabus B. Subsequently, syllabus C was implemented in the early 1970s in the secondary schools and lasted a decade until syllabus D came along. In 1979 the New Educational System was implemented in Singapore and the Curriculum Development Institute of Singapore (CDIS) was entrusted with the task of publishing school textbooks. The CDIS is now called the Curriculum Planning and Development Division (CPDD) and since 2001 textbook writing and publication has been privatized. By the end of the 1980s, changes in the school mathematics curriculum had roughly followed a ten-year cycle. The policy makers were quick to respond to the dismal performance of Singaporean students, ranked 16th, in the Second International Science Study (see Ng, 2001). In tune with changes taking place in mathematics and science education around the world, it was the right

moment for Singapore to launch a major change in science and in school mathematics. Thus, the Singapore Mathematics Curriculum Framework (SMCF) was born.

Why Examine the SMCF?

First, Singaporean students at grade 8 level were the top-performers in the world in each of the three major studies carried out by The *International Association for the Evaluation of Educational Achievement* (IEA) from 1995 to 2003. The first of these studies conducted in 1995 was called the *Third International Mathematics and Science Study* (TIMSS-95). TIMSS-95 involved 45 countries, each of which was tested in both mathematics and science at grade 3, 4, 7, and 8. Students from Singapore ranked 2nd at grade 3 and first at grades 4, 7, and 8 in mathematics. In 1999, TIMSS (TIMSS-99) was repeated but this time only at grade 8. Of the 38 countries that were involved, students from Singapore ranked first in mathematics again. In 2003, the test was renamed *Trends in International Mathematics and Science Study* (TIMSS-2003). Of the 49 countries and 4 other participants (states and provinces), students from Singapore topped the list again at both grades 4 (score 594) and 8 (score 605) in mathematics. Are these results fortuitous? If not, what explains the excellent performance of students from Singapore? Since children can be assumed to be more or less the same everywhere as far as intelligence is concerned, any difference in performance may probably be attributed to the educational system in place and in this particular case in the mathematics curriculum used in the country.

Second, Singapore is one of the few countries in the TIMSS study that uses differentiated curricula for different groups of students, while most countries around the world provide the same curriculum in mathematics to all students. The only three other countries in the TIMSS-2003 study using differentiated curricula for students of different ability levels are Belgium (Flemish), The Netherlands and the Russian Federation. It is interesting to note that students in these three countries have performed significantly above the international average at both grades 4 and 8. In fact, students from Belgium (Flemish), Netherlands and the Russian Federation scored 551, 540, and 532 and were ranked 5th, 6th and 9th respectively at grade 4 level. At the grade 8 level, they scored 537, 536 and 508 were ranked 6th, 7th and 12th respectively. Does this imply that a differentiated curriculum for students of different ability helps them to perform well in the TIMSS study? The data seem to support this claim. There is no doubt that differentiated curricula provide a more focused experience in mathematics learning to the students in Singapore.

Third, the mathematics curriculum in Singapore is closely monitored and implemented in well-resourced schools by highly trained teachers most of whom are subject specialists. Besides, Singapore has a centralized system of education and as such has a national curriculum for mathematics. The implemented Singaporean mathematics curriculum is highly influenced by various forms of testing and high stakes examinations. Accordingly, the Scheme of Work (SOW) in any Singaporean school strongly reflects a substantial part of the intended mathematics curriculum and a strong commitment from the school to implement it. This also allows for a close monitoring of the mathematics curriculum. The Ministry of Education Report (MOE, 2000) for TIMSS-99 highlighted the following methods that are used to support or monitor the curriculum implementation: pre-service and in-service teacher education; mandated or recommended textbooks; instructional or pedagogical guides; achievement standards; directives and notes from the ministry; and school inspection or audit. These practices may not be unique to Singapore. Other

countries also have similar practices. However, Singapore probably has an edge on the overall monitoring process compared to the other countries because its small size allows the country to exercise greater control on the whole educational system. Directives and policies from the ministry are disseminated quickly and in a highly efficient manner. On the other hand, feedback on the implementation process from the schools is quick to reach the ministry.

Following the TIMSS-2003 results (see Mullis, Martin, Gonzalez & Chrostowski, 2004), the Ministry of Education in Singapore (2005) identified the following possible contributing factors: (1) students attitudes towards mathematics; (2) students' educational aspirations and home resources; (3) perceptions of school climate; (4) availability of school resources; and (5) safety in schools. Attributing the success of Singaporean students in TIMSS-2003 to only one of the above factors or to any collection of these factors is not enough. There are probably a host of other factors, some highlighted in the TIMSS study, and some which are not. The logical other place to look for any explanations regarding this matter is the Singapore Mathematics Curriculum Framework (SMCF).

The Singapore Mathematics Curriculum Framework (SMCF)

The Pentagon Model of the Singapore Mathematics Curriculum Framework (SMCF) was developed by MOE and first published in 1990 against a backdrop of changes in mathematics curricula in many parts of the world. For example, in India, 1986 marked the introduction of the National Curriculum Policy while a national curriculum was also adopted in the United Kingdom in 1988. In the United States, the National Council of Teachers of Mathematics (NCTM) published the *Curriculum and Evaluation Standards for School Mathematics* in 1989 and 1990 marked proposals for a new vision of mathematics education in Australia as well. Although there have been a few changes, the core aspects of the SMCF (Figure 1) have survived and can be considered as having been an effective model for mathematics instruction in schools, as highlighted by the performance of Singaporean students in international mathematics achievement tests. The curriculum document can be downloaded from the ministry's website: www.moe.gov.sg. At the core of the model is problem solving and the five sides forming the pentagon are: concepts, processes, metacognition, attitudes, and skills. The pentagon model clearly goes beyond the simple enumeration of content for the school mathematics syllabus.

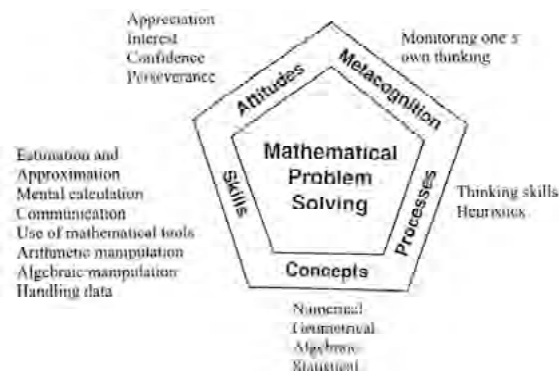


Figure 1. The Singapore Mathematics Curriculum Framework — Pentagon Model

It emphasises not only the content to be taught but also the processes and affective aspects of learning mathematics. Thus, Singapore had a curriculum model developed to suit its own specific needs and there was some local pride attached to it. The SMCF has received some minor refinements over time. From 1997 three initiatives from the Ministry of Education (MOE) were introduced — Thinking Schools and Learning Nation (TSLN), Information Technology (IT), and National Education (NE), were incorporated in the curriculum. In 1998, the mathematics syllabus underwent a content trimming exercise (Kaur, 2003); however the Pentagon Model still remained the backbone of the school mathematics curriculum in Singapore. The top-level performance of Singaporean students in the TIMSS studies from 1995 onwards can be considered as a direct consequence of the changes made in the curriculum through the SMCF.

Three aspects of the curriculum were highlighted by Robitaille and Dirks (1982) which were later used as a model in the Second International Mathematics Study (SIMS): the intended curriculum, the implemented curriculum, and the attained curriculum. In what follows, the SMCF will be examined in the light of these three aspects of the curriculum.

Intended Mathematics Curriculum

All schools in Singapore follow the SMCF guidelines for teaching mathematics. There is a curriculum differentiation for different groups of students, based on their ability. The SMCF details out the aims of teaching the subject and as well the content through which to attain the aims. In the post-TIMSS-2003 period, comparing the SMCF with similar ones in the United States, the American Institutes for Research (AIR, 2005) study claimed that the SMCF was a mathematically logical and uniform national framework that develops topics in-depth at each grade level. The AIR study also highlighted that the SMCF at primary level, included a relatively small number of topics which are carefully sequenced grade-by-grade in a spiral curriculum, in which the topics that are revisited at a more advanced level later.

Besides problem solving, there is an emphasis in the SMCF on affective issues (appreciation, interest, confidence, and perseverance) in learning the subject. Curriculum materials, such as textbooks, are based on the SMCF and are now developed by private companies who have to get MOE's approval prior to the sale of these textbooks to schools. The AIR study (AIR, 2005) was also very positive about the Singaporean textbooks. It claimed that Singapore's textbooks build deep understanding of mathematical concepts through multistep problems and concrete illustrations that demonstrate how abstract mathematical concepts are used to solve problems from different perspectives. The AIR study highlighted the general concrete to pictorial to abstract approach (CPA) in the Singaporean mathematics textbooks. Students in Singapore are expected to solve quite hard problems right from the primary level. For example, it is expected that students at the primary level solve the following types of problems without the formal use of variables.

1. The participants in a competition are P5 and P6 pupils in the ratio of 2: 1. All the P5 participants are girls. Among the P6 participants, the ratio of girls to boy is 4: 3. There are 30 more P5 girls than P6 girls taking part in the competition. How many participants in the competition are girls? (PSLE)
2. Father gave Jason 50 stamps. 16 % of them were from Australia. After Mary gave Jason some more stamps from Australia, the percentage of his stamps which were from Australia increased to 30%. How many stamps did Mary give Jason? (PSLE)

Students in Singapore use the famous *Model Method*, which is basically a pre-algebraic method using models, to solve such problems. Thus, the intended mathematics curriculum places a strong emphasis on problem solving at a very young age.

Implemented Mathematics Curriculum

Singapore is a very small country and has a centralized system which is more manageable when compared to the decentralized systems of large countries such as Australia and the United States. Individual schools develop their Scheme of Work (SOW) based on the SMCF. At the primary level, the implemented curriculum for Primary 1 to Primary 4 is the same. In Primary 5 and Primary 6, two slightly different curricula are implemented: the EM1/EM2 and the EM3. The EM3 syllabus is generally reserved for weaker students and is a subset of the EM1/EM2 syllabus. At the secondary level, the different streams (Special, Express, Normal Academic, Normal Technical) get differentiated curricula based on the ability level of the streams. The mathematics curriculum at the secondary level is in line with the O-Level syllabus of the University of Cambridge Local Examinations Syndicate. Special and Express stream students take four years whereas Normal Academic stream students take five years to finish secondary schooling. Some selected schools have started the Integrated Programme (IP), in which students do not take O-Levels but go directly to do the pre-University level.

As for textbooks, individual schools at primary and secondary can choose one of the MOE-approved series. Schools' SOWs are based on the outline of the content within these textbooks with minor variations between schools in the sequence of presentation of individual topics. At the Junior College level (JC), students can opt to do any one of two integrated mathematics courses. They can do either Syllabus C only, the regular A-Level syllabus or choose to do a more advanced subject called Further Mathematics. There are no prescribed textbooks or curriculum development at the JC level. Teachers in individual JCs develop their own teaching materials.

The implementation of any curriculum depends on the resources that are available within a country. In Singapore, the curriculum is implemented in very well-resourced schools - highest rating for availability of resources among all participating nations in TIMSS-2003 - by highly trained teachers who are subject specialists. The AIR study (AIR, 2005) stressed that Singaporean elementary teachers are required to demonstrate mathematics skills superior to those of their U.S. counterparts before they begin teacher training. At every phase of pre- and post-service training, they receive better instruction both in mathematics content and in mathematics pedagogy. After a content-driven pre-service preparation, Singaporean teachers continue to improve their knowledge and skills through 100 hours of required annual professional training.

Also the school climate in Singapore is perceived to be very safe by both teachers and students (see Mullis et al., 2004). There is a worksheet culture in the country, whereby teachers tend to reinterpret the curriculum, within the constraints of the SOW, by preparing teaching materials more suited to the specific needs of their respective classes. Another important factor that should not be overlooked is the teaching time for mathematics out of total teaching time (22% at grade 4 and 15 % at grade 8).

Attained Mathematics Curriculum

The attained curriculum is what students demonstrate as having learned through the schooling process. It is not only decided by the grades of students but also by the attitudes and other affective characteristics that students develop through the curriculum. One of the possible ways to know about what is attained is to use appropriate assessment procedures. Comprehensive assessment of learning of mathematics may be very complex. In countries such as the United States, various school assessments, examinations and standardized tests provide an indication of what has been attained or learned by the students. Singapore has different assessment and testing practices. School-based assessments contribute very little to final certification at the end of the primary, secondary or JC cycles. High stakes examinations such as the Primary School Leaving Education (PSLE) at primary level; O-Levels at the secondary level; and A-Levels at the JC level decide students' levels of curricular attainment. The questions on Singapore's high-stakes grade 6 Primary School Leaving Certificate (PSLE) are more challenging than the released items on the U.S. grade 8 *National Assessment of Educational Progress* and the items on the grade 8 state assessment tests (AIR, 2005). Singapore's most challenging questions are designed to help Singapore identify the best students. As a way to hold schools responsible as well as students accountable for performance, Singapore uses a yearly measure of each school's value-added contribution to students' achievement. Schools that perform above expectations are recognized and rewarded.

An overview of the performance of students in international achievement tests may also give an understanding of the attainment levels of students in Singapore. However, any international testing may not adequately represent the actual level of attainment of students in a country. As such, the results of these studies have to be interpreted with caution. In addition, Singapore is one of the few countries in TIMSS-2003 with significant gender differences at both grades 4 and 8. The gender differences are also significant in three content domains (Number, Algebra, and Geometry) in mathematics at Grade 8 level. Unlike Singapore, there are no significant gender differences in the overall scores for students from the other top-performing Asian countries, although there were no significant gender differences in the earlier TIMSS studies. It should be noted that Singaporean students have showed positive attitudes towards the learning of mathematics, unlike students from other top-performing Asian countries in TIMSS-2003.

Perceived Strengths and Weaknesses of the SMCF

The SMCF should be examined not only in the light of performance in the TIMSS studies but also in the context of a very specific socio-cultural context which places strong premium on education. As any other mathematics curriculum in the world, the SMCF demonstrates some strength and also some weaknesses. It is not the ultimate curriculum and it is not simply pro-TIMSS. However, it is a fact that students using the SMCF have done well in TIMSS. For example, based on the TIMSS-2003 study data (see Mullis et al., 2004), the following factors, connected to the SMCF, seem to stand out as possible contributing factors to the top performance of grade 8 students from Singapore:

1. A high proportion of students with a positive attitude to mathematics. At least 75% of students reported enjoying the learning of mathematics a little.
2. A high index of self-confidence in learning mathematics among Singaporean students

(39% in the High category).

3. A high value attached to mathematics by students (63% in the High category).
4. High educational aspirations (56% expect to finish university)
5. A larger amount of time devoted to homework (89% in the Medium and High categories).
6. High availability of school resources (88% of students in the High category).
7. School perceived to be safe by 91% of the teachers
8. A differentiated curriculum to match the ability of students.
9. A high percentage of coverage of the TIMSS topics (80%) in the school curriculum.
10. A close monitoring of the mathematics curriculum by the ministry.

These factors are not necessarily unique to Singapore. Many of them are shared with other countries and, in particular, with the top-performing Asian countries. However, what seem to stand out from these listed factors are: (1) the affective aspects of students — positive attitude, strong self-confidence, and high value attached to mathematics; (2) a differentiated curriculum to match different ability levels. For a country where only 23% of the students always speak the language of the test at home, the Singaporean students' performance in TIMSS is highly commendable.

A point to be noted is that, although the SMCF places a significant emphasis on geometry, the overall performance in geometry of Singaporean students has been below the country's average performance in other content domains in each of the three TIMSS studies from 1995 to 2003 at grade 8 level. However, the performance of Singaporean students is still well above the international average. A few changes will probably be made to the geometry topics in the SMCF.

Given the fact that Singapore has been a top-performer in the TIMSS studies, a lot of attention has been focused on the country's educational system. Following the TIMSS-99 study, a group American educators interested in comparing textbooks from Singapore with similar ones in the United States, visited Singapore. They drafted a report for the National Science Foundation (NSF). They found the textbooks from Singapore as being very traditional, with lots of practice questions for students, to be served as a reference book under the authority of a traditional teacher (NSF, 2001). This is quite different from the conclusions of the AIR study (AIR, 2005). Although the NSF report claimed that the quality of and accuracy of the mathematics in Singapore's textbooks is high, it also added that:

While the mathematics in Singapore's curriculum may be considered rigorous, we noticed that it does not often engage students in higher-order thinking skills...Singapore's students are rarely, if ever, asked to analyze, reflect, critique, develop, synthesize, or explain. The vast majority of tasks in the Singapore curriculum is based on computation, which primarily reinforces only the recall of facts and procedures...but we feel it discourages students from becoming independent learners (p. 8).

This type of characterization of the Singaporean curriculum is quite common. The AIR study (2005) also mentioned the lack of higher order thinking skills in the Singaporean curriculum. However, it is difficult to reconcile with the fact that these very same students coming out of a system, which supposedly does not encourage *higher order thinking*, are the top-performers in the world in mathematical performance. The TIMSS tests are not supposed to be testing rote memory, recall of facts and manipulation of skills. The authors of the NSF report were quick to add that:

However, the TIMSS tests were designed to test understanding of concepts in addition to competency. The inferences that can be made thus become more murky. On one hand we must acknowledge that Singapore's educational system — the curriculum, the teachers, the parental support, the social culture, and the strong government support of education-has succeeded in producing students who as a whole understand mathematics at a higher level, and perform with more competence and fluency, than the American students who took the tests (p. 51).

Some arguments go beyond the simple bounds of the curriculum. For example Brown (1999) has claimed "Singapore and Taiwan seem to achieve good results using traditional transmission teaching of a rather boring sort" (p. 201). Furthermore, Brown added that pupils in the Pacific Rim countries are urged by their teachers to do well to prove their country is superior because nationalism is an important element of society. There have been other criticisms directed at the SMCF which need careful examination. For example, amongst others, there have been questions about inadequate use of technology, lack of probability and statistics topics in early grades, unbalanced gender references, and inadequate recognition of students with varying learning styles.

In conclusion, it can be said that the SMCF has proved to be a robust curriculum for Singapore. However, it should be not taken as a model of mathematics curriculum in other countries. The specific socio-cultural context within which the SMCF operates should be given due attention. Also, no claim has been made about the TIMSS studies in this paper. TIMSS studies may have their weaknesses but they also have much of value to offer as claimed by Howson (1999). One must be guarded against using the SMCF as a model for TIMSS. The SMCF still needs refinements. In an attempt to improve the SMCF, some changes are forthcoming. However, these changes will not affect the core of the curriculum.

References

- American Institutes for Research. (2005). *What the United States can learn from Singapore's world-class mathematics system (and what Singapore can learn from the United States): An exploratory study*. Washington, DC: AIR
- Brown, M. (1999). Problems of interpreting international comparative data. In B. Jaworski & D. Phillips (Eds.), *Comparing standards internationally: Research in mathematics and beyond* (pp. 183-205). Oxford, UK: Symposium Books.
- Howson, A. G. (1999). The value of comparative studies. In G. Kaiser, E. Luna, & I. Huntley (Eds.), *International comparisons in mathematics education* (pp. 165-188). London, UK: Falmer Press.
- Kaur, B. (2003). Mathematics for all but more mathematics for some: A look at Singapore's school mathematics curriculum. In B. Clarke, A. Bishop, R. Cameron, H. Forgasz, T. S. Wee (Eds.), *Proceedings of the MAV Annual Conference* (pp.440-455), 4-5 December, 2003.
- Lee, P. Y., & Fan, L. (2002). *The development of Singapore mathematics curriculum — understanding the changes in syllabus, textbooks and approaches*. A talk given at the Chongqing conference 17-20 August 2002. Unpublished.
- Ministry of Education Singapore. (2000). *Third International Mathematics and Science Study 1999 (TIMSS 1999): National Report for Singapore*.
- Ministry of Education Singapore. (2005). *Trends in International Mathematics and Science Study (TIMSS) 2003: National Report for Singapore*.
- Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report*. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre.
- National Science Foundation. (2001). *Middle school mathematics comparisons for Singapore Mathematics, Connected Mathematics Program, and Mathematics in Context: A summary*.
- Ng, S. F. (2001). The Singapore story: A nation's effort to develop human resources. *NCTM Mathematics Education Dialogues*.
- Robitaille, D., & Dirks, M. (1982). Models for the mathematics curriculum. *For the Learning of Mathematics*, 2(3), 3-21.

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Singaporean students'™ top-level performance in the TIMSS studies in mathematics has strong connections to the mathematics curriculum in the country. While mathematics curricula tend to be somewhat similar in countries all over the world, there are some subtle differences in the Singaporean mathematics curriculum, which are worth examining. The Singapore math method is focused on mastery, which is achieved through intentional sequencing of concepts. Some of the key features of the approach include the CPA (Concrete, Pictorial, Abstract) progression, number bonds, bar modeling, and mental math. Instead of pushing through rote memorization, students learn to think mathematically and rely on the depth of knowledge gained in previous lessons. An attitude that math is important and approachable is also essential. Students perform at a higher level when their potential for understanding and success is assumed. Yes, Singapore has one of the best Math education in the world! Singapore Mathematics teaches each student to learn and master fewer mathematical concepts at greater detail as well as having them learn these concepts using a three-step learning pr... This is followed by drawing pictorial representations of mathematical concepts. Students then solve mathematical problems in an abstract way by using numbers and symbols. Each semester-level Singapore math textbook builds upon prior knowledge and skills, with students mastering them before moving on to the next grade. Students, therefore, need not re-learn these skills at the next grade level.