Review of the convergence of some Krylov subspaces methods for solving linear systems of equations with one or several right hand sides

Hassane Sadok[1], Mohammed Bellalij[2]

[1] LMPA, Université du Littoral,50, rue F. Buisson, BP 699, 62228 CALAIS-Cedex, FRANCE A
[2] Université de Valenciennes, Le Mont Houy, F-59313 Valenciennes Cedex, FRANCE.

Abstract

Krylov subspace methods are widely used for the iterative solution of a large variety of linear systems of equations with one or several right hand sides.

In this talk, we will derive new bounds for the GMRES method of Saad and Schultz, for solving linear system. We will give a sample formula for the norm of the residual of GMRES based on the eigenvalue decomposition of the matrix and the right hand side. This formula allows us to generalize the well known result on the convergence behavior of GMRES when the matrix has a full set of eigenvectors. The explicit formula of the residual norm of the GMRES when applied to normal matrix, which is a rational function, is given in terms of eigenvalues and of the components of the eigenvector decomposition of the initial residual. By minimizing this rational function over a convex subset, we obtain the sharp bound of the residual norm of the GMRES method applied to normal matrix, even if the spectrum contains complex eigenvalues. Hence we completely characterize the worst case GMRES for normal matrices. We use techniques from constrained optimization rather than solving the classical min-max problem (problem in polynomial approximation theory)

Known as one of the best iterative methods for solving symmetric positive definite linear systems, CG generates as FOM an Hessenberg matrix which is symmetric then triangular. This specific structure may be really helpful to understand how does behave the convergence of the conjugate gradient method and its study gives an interesting alternative to Chebyshev polynomials. The talk will deals also about some new bounds on residual norms and error A-norms using essentially the condition number. We will show how to derive a bound of the A- norm of the error by solving a constrained optimization problem using Lagrange multipliers.

References

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The convergence problem of Krylov subspace methods, e.g. FOM, GMRES, GCR and many others, for solving large unsymmetric linear systems has been intensively investigated. There are many results in the literature, mainly for the case where the coefficient matrix A is diagonalizable and its spectrum lies in the open right (left) half plane. In this paper, we focus on a convergence analysis of those Krylov subspace methods which give rise to a similar minimax problem in the case where the coefficient matrix A is defective. When the spectrum of A either lies in the open right (left) half plane or i Krylov subspace methods are widely used for the iterative solution of a large variety of linear sys-tems of equations with one or several right hand sides. In this talk, we will derive new bounds for the GMRES method of Saad and Schultz, for solving linear system. We will give a sample formula for the norm of the residual of GMRES based on the eigenvalue decomposition of the matrix and the right hand side. This formula allows us to generalize the well known result on the convergence behavior of GMRES when the matrix has a full set of eigenvectors. I'm currently working on a fluid simulation in C++, and part of the algorithm is to solve a sparse system of linear equations. People recommended using the library Eigen for this. I decided to test it out using this short program that I wrote: #include #. include #include . int main() { std::vector> triplets; triplets.push_back(Eigen::Triplet(0, 0, 1)); triplets.push_back(Eigen::Triplet(0, 1, -2)); triplets.push_back(Eigen::Triplet(1, 0, 3)); triplets.push_back(Eigen::Triplet(. 1, 1, -2)) linear-algebra convergence krylov-method conjugate-gradient. Share.Â The point of Krylov methods is to accelerate (or even enforce) the convergence of a given stationary linear iteration. $\endgroup$ â€“ Thomas Klimpel Jan 29 '12 at 21:40. 5. $\begingroup$ A paper that I think was written to answer your questions is Ipsen and Meyer, The idea behind Krylov methods, Amer.Â Krylov methods subspace methods are in essence projection methods: You pick subspaces $U,V\subset \mathbb{C}^n$ and look for a $\tilde x \in U$ such that the residual $b-A\tilde x$ is orthogonal to V. For Krylov methods, U of course is the space spanned by powers of A applied to an initial residual.