Differentiating Instruction with Marbles: Is This Algebra or What?

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"If the only tool you have is a hammer, everything begins to look like a nail."
--Mark Twain.

Abstract

Our inquiry began with a problem-solving lesson designed to facilitate pre-service teachers' making meaning of how some algebraic concepts could be understood by children long before the traditional middle-school algebra class. Follow-up experiences with many groups of practicing and pre-service teachers revealed some patterns.

Initially, resistance was encountered with many teachers describing their dislike or inability to "do" algebra. As we scaffolded our instruction to meet their individual levels of understanding, teachers were able to communicate a variety of activities geared toward teaching for understanding, addressing students at different developmental levels through their own differentiation of instruction.

Teacher reflections were facilitated along three dimensions: (a) the metacognitive process of learning algebra as well as teaching strategies, (b) applying the pedagogical content knowledge gained in their own classrooms, and (c) designing and implementing differentiated instruction to benefit all students in their classrooms. This study is a self-reflective inquiry into these experiences.

Introduction

Standards and how we assess them are at the center stage at this time in most discussions in and about the education. Those discussions revolve around how and in what specific ways teachers and students will be affected. For example, how and shall we free teachers of the burden to cover all of the curricular materials as they differentiate instruction, facilitating learning of how to learn and think in a particular subject area? How will we develop assessment of students that is more focused on how well they can demonstrate their ability to use strategies to learn for understanding rather than the mastery of specific facts or concepts alone? How would those changes impact standardized assessments?

No matter what standards we are talking about, knowledge and skill do not guarantee understanding (Perkins, 1993). If acquired knowledge and skills are not understood, their active use and application are not to be expected. Teaching for understanding also has strong support from cognitive science, educational psychology, and our practical experiences with teachers and students (Gardner, 1991; Perkins, 1986, 1992). Langer (2000) found that teachers in higher performing schools go beyond students' acquisition of skills or knowledge to engage them in creative and critical uses of the skills and knowledge covered.

Research based fundamental revisions in the Piagetian conception of developmental factors suggest that children in supportive environments can understand much more than what was previously thought. Complexity is a critical variable because younger children cannot readily understand concepts that involve multiple sources of variation at once. There are some essential elements in that process that should be nurtured on an ongoing basis as students move from pre-kindergarten through elementary, intermediate, middle and high school. When we teach for understanding, differentiation of instruction is essential. We recognize the support for this idea in the *Principles and Standards for School Mathematics* where the expectations are that teachers make provisions for *all* students (NCTM, 2000).

Without differentiation, many students cannot benefit from instruction in their general education curriculum. For students with disabilities included in the general education classroom teachers have always shared responsibility for educating them. Their role changed significantly however, with the reauthorization of the Individuals with Disabilities Education Act in 1997. The specially designed instruction that defines special education is no longer the sole responsibility of special educators delivering services in segregated settings. Content, methodology, and delivery of instruction must now be adapted to the needs of students with disabilities to ensure access to the general curriculum so that they can meet the educational standards that apply to all children (IDEA, 1997).

In the context of differentiating instruction to teach for understanding, both teachers and students confront a variety of difficulties related to specific areas of content. In algebra, they range from poor arithmetic skills and an inability to apply their knowledge to new problems, to difficulty with the abstract content. Edwards (2000) refers to algebra as a “gatekeeper" to the study of other academic fields. He also points out that middle school teachers are under pressure not only regarding the ways they teach algebraic concepts but also to facilitate students’ learning of mathematical concepts in ways that will support their formal study of algebra in the future.

Supporting the approach for identifying the "big ideas" of algebra that are appropriate to middle school (Edwards, 2000), we report here about our experiences in extending some of those ideas in developmentally appropriate ways through all the grade-bands (preK-2, 3-5, 6-8) and ranges of ability. The initial goal was to model for preservice teachers activities and lessons that can be carried throughout different grades so that a future algebra student is developing an evolving idea of systems of algebraic equations and other related underlying concepts and principles over time.

The cumulative effect of student difficulties in transition between arithmetic and algebraic concepts is a gap that is not easily bridged. Difficulty at the abstract level might mean that students struggle to solve problems using symbolic representations; they may need pictorial representations or hands-on manipulatives. Students need continued arithmetic assistance and/or more concrete methods with algebra concepts throughout their academic career (Witzel et al., 2001).

The “seed” for this study came out of an open-ended problem-solving lesson conducted with preservice preK-8 teachers in an instructional strategies class. The initial intent was to demonstrate an algebra activity that has something to offer for every developmental level. We have been facilitating variations of this activity with other groups of preservice teachers, graduate students and in professional development workshops for practicing teachers (henceforth referred to as teachers). Many of these teachers facilitated these types of activities in their classrooms and reported about their experiences. In every new encounter with this activity we learned something novel: about teachers as diverse learners, about the significance of reflective practice, about conceptual understanding of algebraic ideas, and about not underestimating what teachers/students can accomplish with experienced guidance.

The purpose of the study was twofold: (a) to explore elementary teachers' understanding of the sequence of concepts and ideas necessary to make sense of algebraic system(s) of equations (understanding algebra); and (b) developing algebra related pedagogical content knowledge (teaching algebra preK-8). Considering the extreme variance in understanding algebraic concepts of teachers and the broad spectrum of their potential audiences, the decision making process was guided by one of the underlying essential principles of our teaching philosophy: *differentiating instruction for individualized learning*.

In researching a common language to communicate our experiences, we recognized formal structures in the processes that support teachers in their individual learning efforts. Guiding teachers in what to consider (conceptual scaffolding) and in ways to think (metacognitive scaffolding) became an underlying grid of the instructional design. These conceptual and metacognitive scaffolding tools were employed extensively during this process. Teachers recognized the value of investing time in a single learning activity that addresses many outcomes in depth, an approach that allows all students to benefit from instruction. The use of metacognitive scaffolding became a driving force in teaching teachers how to differentiate.

Along with activities, teacher reflections were facilitated along two dimensions: their metacognitive process of learning (algebra) and their thinking about the development and application of the algebra-related pedagogical content knowledge. Inseparably, we report on our own reflective cycles that guided instructional decision-making.

Theoretical Framework: Teaching Algebra for Understanding to All Children

We consider the following components of the theoretical framework underpinning our teaching philosophy: understanding as flexible performance (Perkins, 1993), differentiating instruction for individualized learning (Postlethwaite, 1993), and all students are capable of learning algebra. These components are closely reflected in the researchers' beliefs about teaching and learning algebra for all children and therefore are fundamental in considering the context for this research.

When students attain understanding, what have they achieved? (Perkins, 1993)

What a student does in response to the questions that put understanding into action shows their level of understanding. The student might be able to solve an equation, but if there is no understanding of where the equation is coming from or where and how to use it, they may just be using a memorized skill that is going to be useful only for that type of equation, nothing more. Understanding a concept or a topic of study is being able to carry out a variety of actions or performances with the topic by the ways of critical thinking: explain, apply, transfer, generalize, represent in a new way, make analogies and metaphors, and so on. It is being able to take knowledge and use it in new ways (Perkins 1993).

General guidelines lead teachers to recognize priorities when teaching for understanding: What shall we teach? What is worth understanding? How shall we teach for understanding? How can both students and the teacher know what students understand and how can they develop deeper understanding? Perkins (1993) identified some of these priorities: (a) facilitating learning as a long-term, thinking-centered process; (b) providing rich ongoing assessment with supportive feedback and opportunities for reflection; (c) supporting learning with powerful representations; (d) recognizing flexible conceptions of what students can and cannot learn at certain ages; and (e) requiring the learner to carry facts and principles they acquire in one context into other contexts. Inducting students into mathematics as a system of thought that recognizes understanding in the nature of mathematics is embedded within these priorities.

All students can learn and succeed, but not on the same day in the same way.

–William Spady

Though district or state or national standards may not be achieved at the same rate or on the same day for every student, the goal is for all students to make progress within the curriculum to the greatest extent they are capable. With the introduction of inclusion, the diversity of learners within each classroom becomes even greater. For all students to be actively engaged and making progress within the curriculum, teachers need to be differentiating instruction as part of their ongoing preparation for instruction and the instruction itself. In the process of differentiating instruction, teachers must be sure to challenge students regardless of their present level of academic performance (Postlethwaite, 1993).

In the differentiated classroom, individual student growth is emphasized over relative standing among classmates. How do we implement a standards-based curriculum without standardizing the instruction for all learners? The core of *what* students must learn remains consistent and the diverse nature of students remains consistent. What the classroom teacher must vary is *how* they guide students through constructing and demonstrating understanding of the knowledge, skills, and attitudes prescribed in those standards. The *how* is best practice in teaching, instruction that meets the diverse learning needs of all students and can be achieved through differentiating instruction.

Designing classroom environments for differentiated instruction seems like a challenging task. It is overwhelming to plan for varying the levels of difficulty, modes of expression, degrees of scaffolding, complexity of tasks, grouping arrangements, and a host of other variables for each student and activity. Consider the teacher variables involved: experience, content knowledge, management skills, tolerance for varying levels of student activity, and self-efficacy for teaching a heterogeneous group of learners. Such complexity may be minimized when differentiation is based on student need lest you end up standardizing your teaching practices and lose sight of your primary objective: personalized learning (Cole et al., 1994; Friend & Bursuck, 1999).

All Students Should Learn Algebra - When And Where Does It Start?

The a*lgebra standard* for school mathematics (NCTM, 2000) states that instructional programs from *prekindegarten through grade 12* should enable students to: (a) understand patterns, relations, and functions; (b) represent and analyze mathematical situations and structures using algebraic symbols; (c) use mathematical models to represent and understand quantitative relationships; and (d) analyze change in various contexts. By viewing algebra as a strand in the curriculum from pre-kindergarten on, teachers can help students build a solid foundation of understanding as a preparation for more sophisticated work in algebra in the middle grades and high school. Initially, students are classifying and ordering which are natural and interesting for young children. Later, they may describe the regularity in patterns verbally rather than with mathematical symbols (English & Warren 1998). In grades three to five, they can begin to use variables and algebraic expressions as they describe and extend patterns. In the middle grades, students should focus on understanding linear relationships.

Algebra is more than moving symbols around. Students need to understand the concepts of algebra, how manipulation of the symbols is governed, and how the symbols themselves can be used for recording ideas and gaining insights into situations. For example, systematic experiences with placeholders and numbers and their properties lay a foundation for later work with symbols and algebraic expressions. If students engage extensively in symbolic manipulation before they develop a solid conceptual foundation for their work, they will be unable to do more than mechanical manipulations (NRC, 1998). For meaningful work with symbolic notation, the foundation should be laid over an extended period of time (K-8). Experiences leading to such understandings should be ongoing.

In addition, algebra is about abstract structures and using the principles of those structures in solving problems expressed with symbols. By learning that situations often can be described using mathematics, students can begin to form elementary notions of mathematical modeling. Research also indicates a variety of student difficulties with the concept of a variable (Küchemann, 1978; Kieran, 1981) so, developing a thorough understanding of a variable over the grades is important and it needs to be grounded in extensive experience (Sfard, 1991).

The ideas included in the algebra standard constitute a major component of the school mathematics curriculum and help to unify it. *Kids with Marbles* is a rich activity that has potential to show an evolving approach that establishes roots for the understanding of algebra concepts. The activity appeared to be more fruitful than expected and resulted in a rich source for ongoing discussion about developmentally appropriate mathematics activities. It initiated broad discussions and many questions, both mathematical and pedagogical. From concrete hands-on ‘guess and check’ to formal algebra methods, teachers used a *variety of strategies* for solving a system of five equations with four variables.

Research in Context: Naturalistic Paradigm Components

This study seeks to make meaning through multiple realities (Erlandson et al., 1993) of different groups of teachers and the two researchers (ourselves), in an effort to understand what works well in terms of guiding teachers in sense-making of: (a) algebraic concepts leading to an understanding of systems of equations, and (b) differentiation of instruction using conceptual and metacognitive scaffolding tools. Identifying a variety of constructions through whole class and small group discussions, teacher and researcher reflections, we try to bring into "as much consensus as possible" a variety of existing understandings around this theme (Guba, 1990, p.26).

The model used for this study adopts principles of the naturalistic research paradigm (Erlandson et al., 1993; Lincoln and Guba, 1985; Moschkovich & Brenner, 2000), combining naturalistic and cognitive science methods for collecting and analyzing data. It is similar to the anthropological one used by Moschkovich (see, Moschkovich & Brenner, 2000, p.467) (making sense of linear function). His study focused on students' cognition in a school setting and this study is focusing on pre-service and inservice teachers' cognition of learning to teach algebra for understanding. The project included the collection of data from a number of settings (preservice methods classes, workshops for practicing teachers, special education graduate methods class), classroom observations, written assessments and reflective journaling about both the process and content.

This research started with observations in a setting where preK-8 preservice teachers learn methods for teaching mathematics. Students were observed working in groups and during class discussions in five successive semesters. The researchers participated in class discussions guided with questions that are reported in the section "How Did It Go?"

Data are collected and used for both ongoing design of learning activities by the instructors and ongoing research about preservice elementary teachers' beliefs about teaching mathematics and development of their mathematics related pedagogical content knowledge. The cycle of data collection began by observing students in this natural setting and making conjectures based on these observations. These conjectures play a significant role in designing classroom activities across learning environments (preservice methods classes, workshops for practicing teachers, special education graduate methods class) and data sources (written assessments, class discussions and observations, reflective journaling) and include guided questioning, both metacognitive and conceptual (Glaser & Strauss, 1967).

Four concurrent yet diverse perspectives were reflected in our analysis: One researcher's perspective is informed by training in mathematics and teaching mathematics, the second researcher's perspective is informed by training and teaching in special education, the third is preservice students' perspective and the fourth is that of the practicing teachers. All very different by their experiences, interests and views of teaching and learning, forming an intrinsic web of both understandings and misconceptions to wade through and interlace.

These diverse perspectives resulted in the emergence of additional research questions. These new questions and the strategies used to explore them constituted a spiraling process that added to the knowledge and understanding of everyone involved. We frequently went into a new learning environment with preconceived expectations for what would unfold in terms of teachers' learning and understanding. Occasionally however, we were surprised with the additional questions and representations that emerged as a result of the learners' experiences. As we report on our findings so far, we continue our naturalistic inquiry into the essence of teaching and learning for understanding.

The Process: Understanding/Playing with Marbles and Dominos

We report our research findings about teachers' application of and reflections around these learning experiences. We also analyze our students' reflections and our own role in facilitating the activity that was conducted a number of times with a variety of audiences.

Activity: How Many Marbles Does Each Kid Have?

Each group with 4 students gets an envelope with 5 cards (Erickson, 1989). The "getter" passes one card to every student and keeps two for himself:

1. Anna and Brad have five marbles when you put all their marbles together. How many marbles does each kid have?
2. Anna and Cory have nine marbles when you put theirs together. How many marbles does each kid have?
3. If Cory and Dona put all their marbles together, they'd have fourteen. How many marbles does each kid have?
4. If Dona and Brad put all their marbles in one together, they would have ten marbles. How many marbles does each kid have?

5. If the four kids put all their marbles together and shared them equally, they'd each get four, but there would be three marbles left over.

The usual rules for cooperative learning apply:

- Follow the rules of the activity
- Make sure everybody gets to participate
- Listen to what other people say
- Try to give reasons for what you say
- Ask others for their opinions
- Help others - without telling them what to do or giving answers
- Get help if you need it - from the group first and the teacher last

Two following additional rules for this activity may be used to reinforce verbal/reading communication skills:

- You may look only at your own clue. You may not look at anyone else's.
- You may share your clue by telling others what's on it, but you may not show it to anyone else.

How Did It Go?

Students open their envelopes, read their cards, and take turns to tell others in their group what information/clue is on their cards. Though they are given cooperative learning rules and assignments to follow, one person generally emerges as the group's leader. While students are sharing clues from their cards, the emerging recorder documents the activity (strategies & solutions) of the group.

Groups try different strategies. Typically, one group solves the problem and is elated that not only have they found a solution, but were the first to do so.

"We are done!" They exclaim.

"Are you sure?" We ask.

We ask them to explain how they got their solution and they do. At this point, they believe they are finished, their body language even reflects this as they call us over to show off their solution. We then challenge their sense of "triumph" by asking them "Is there another solution?" or " Can you find another one?"

With a collective sigh and surprised look on their faces they ask, “You mean there’s another one?” Or, jokingly, still puzzled – “Since YOU are asking there must be; I knew it could not be so easy.” Once back at work using the same strategy or attempting another, they do indeed find that there are others solutions, several in fact.

Recognizing the different rate in which groups are working we further facilitate their progress asking, as appropriate, questions from the following sequence:

- Is there more than one?
- Is that all? How do you know? Convince me/your partner.
- How did you come to understanding that there were no more solutions?
- How did you organize your solutions?
- What strategies did you use?
- Did you use manipulatives? If so, did they help?
- Would you share your method?
- Any patterns?

Some groups get one solution right away and given a prompt, seek the others. Other groups need prompts to find one solution then are convinced there are no more. Still other groups are unable to find even one solution until offered the use of manipulatives. Although manipulatives/marbles are always in the room and their use is generally encouraged, rarely does a group use them unless prompted to do so.

For those who finished early we ask – “Give examples of how you could adapt the activity for different grade/developmental levels.”

When all groups have found all solutions, we come together as a whole group/class and all students participate by describing what they learned and experienced by responding to the following questions:

- Will you share your solutions?
- What strategies did you use on this problem? Would you share your method?
- If you used manipulatives, how did they help?
- How did it feel when you found out that the problem had more than one answer?
- What did you do when you got stuck?
- Describe any patterns you identified.

- How did you come to understand there were no more solutions?

We then focus the discussion on how we differentiate instruction for individualized learning. Students were asked to identify how, within their groups, they adapted the activity for students at different grade or developmental levels.

What Strategies Did Teachers Use?

1. Role-playing combined with guess and check
2. Guess and check, trial and error – as a cooperative group
3. Creating a table

Anna	Brad	Cory	Dona
0	5	9	5
1	4	8	6
2	3	7	7
3	2	6	8
4	1	5	9
5	0	4	10

4. Four-pane window (magic square)

Anna	Brad	5
Cory	Dona	14
9	10	**19**

5. Formally solving algebraic system of equations:

$$
\begin{array}{ccccccccc}
A & + & B & & & & & = & 5 \\
A & & & + & C & & & = & 9 \\
 & & & & C & + & D & = & 14 \\
 & & B & & & + & D & = & 10 \\
A & + & B & + & C & + & D & = & 19
\end{array}
$$

Reflections

Teacher reflections were facilitated along three dimensions: (a) the metacognitive process of learning algebra as well as teaching strategies, (b) thinking about applying the pedagogical content knowledge gained in their own classrooms, and (c) designing and implementing differentiated instruction to benefit all students in their classrooms. Teachers communicated their reflections during whole class discussions as well as within their groups. Reflections were also elicited through these follow-up questions:

(a) Why did you stop after finding one solution?

(b) Briefly describe the strategy(ies) your group used to solve *Marbles* problem?

(c) Who was the leader in your group?

In the following section, through our own reflection and interpretation of teachers' reflections, we examine our teaching and learning "from the inside" (Ball, 2000, p. 365). We became aware of the instruction being guided by both our individual and shared reflections of the experience. As instructors, we used their reflection to guide them from knowing to understanding and that meant different approaches (probes, manipulatives, tell me how you know, etc.) for different groups and students within groups.

The Metacognitive Experience: Thinking About Learning

Why did you stop after finding one solution? Each time we facilitated this learning activity, almost every group stopped after finding only one solution. We quote two responses:

It did not even occur to me that there could be other solutions. Again, it was irrelevant, because an answer was found.

We are ***so used to thinking that there can only be one way*** *to find an answer. We are programmed to think that when a strategy results in a correct answer, it is the only one.*

In addition, others responded:

- the directions didn't say anything about more than one possible answer
- we assumed there only was one solution
- we thought all we had to do was find one solution
- we felt there only was one solution

- the problem was solved, did not occur to us there could be other solutions
- we felt we had the answer and were proud that we'd come up with it first

The fact that students stopped after finding one solution goes to the notion of *knowing but not understanding*. Regardless of the strategy, most of the groups found one solution and stopped. If they had developed an understanding of the concept, they would have known there had to be more than one solution, because formally, there were more equations than variables.

Given the responses they provided, it would appear that they have been trained not to think. Just find a solution ("*the* answer") and you are finished. More than that, their attitude appeared defensive as they justified their approach to solving the problem. After analyzing their written reflections, it is our belief that our question, *Why did you stop after finding one solution?* reinforced this defensive stance.

Briefly describe the strategy(ies) your group used to solve Marbles problem? It was a learning experience for us as we observed and analyzed how different groups processed the problem. Discussion was guided by the questions: *What strategies did you use? How did you organize your solutions? Would you share your method?* It became evident that groups were not all at the same level of understanding how to approach this problem. Their *particular level of understanding determined which strategy* they used in solving the problem. Approaches to solving the problem varied from guess & check to formal methods for solving a system of equations, they included role playing, drawing a table, or some combination of strategies. Whatever the strategy, the majority of groups stayed with it until they had found all possible solutions.

When we suggested exploring the use of other strategies, "Can you do it in some other way?" they were reluctant, displaying an attitude that suggested "we're finished, why do more?" Observations of their unwillingness to try different strategies were confirmed by their written reflections. When asked to briefly describe the strategy(ies) used by their group to solve the problem, not a single individual reported considering anything other than the one they used. This reluctance of teachers to think goes to how we differentiate, find different ways and try other things to get students engaged in the learning process. If our teachers are not willing to attempt another strategy, we wondered how they would come to the point of recognizing the necessity to differentiate instruction in this manner. They did not view "trying another strategy" as a way to differentiate. Once we became aware of this our shared reflection guided us in bringing to the surface the use of a variety of strategies as yet another way to differentiate instruction. By doing this we modeled for them how to change direction in midstream when reflection in the midst of instruction indicates the need to do so.

Who was the leader in your group? The nature of the Marbles problem is such that specific roles are not assigned within each cooperative group. We observed however that one person invariable emerged as the group leader. Not only did this person influence the strategy that was selected the strategy seemed to be determined by the level of understanding of this particular leader. This emergence of a leader was confirmed in their written reflections:

> *The leader of the group was Michelle because she directed the solution development.*
>
> *I am not exactly sure what qualifies a person to be the leader. I started off by being the recorder and leading the group by writing the equations we would need to work with. I suggested that we would need some type of table in which to work. Dean soon took charge by working with a table that was more productive than mine. It wasn't long before it became obvious that he knew what he was doing sooner than we did.*

From the Inside: Looking through their lenses

After the activity and follow-up discussion, students were challenged with questions related to how they would differentiate the instruction for their own students. When asked, "What grade do you think this activity is appropriate for?" for the most part, students called out "college or high school algebra class". On occasion someone would suggest 6^{th} or 7^{th} grade. This was invariably followed by a resounding chorus of "No earlier than 6^{th} grade." We then posed the questions:

> *"Assuming this problem is too difficult for the grade level you are teaching, can you make developmentally appropriate modifications that would still incorporate some of the reasoning and thinking that upper grade students would engage in the process of solving this problem?"*

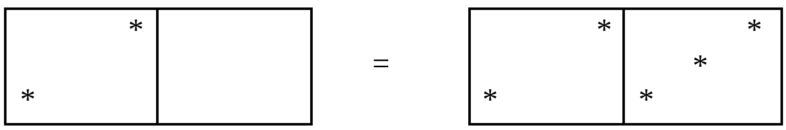
and, in order to focus on teachers' own "look from the inside",

> "*What specific activities could you do in the lower grades that would prepare students to do these types of algebra problems in the middle school grades?*"

One of the major obstacles to understanding algebra is "working out the answer", for example 2+3=__. Students and teachers look at 2+3 and answer "5". It is the one and only answer. However, when we introduce __+__ = 5, we want them to understand that there are many solutions, many ways to represent "5". We want our teachers to think, and therefore teach their students to understand that although 2+3=5 is a fact, there are many ways to represent "5". In this setting, teachers recognized the richness of asking the question in a different way, providing many opportunities for exploration and learning (developing conceptual understanding).

In response to the questions above, teachers came up with many ideas. Here are some examples rank-ordered developmentally:

1. I have two marbles. How many more do I need in order to have 5 altogether? Or, using dominos:



2. Find all dominos with 5 dots. OR, If you and your friend have 5 marbles, how many might each of you have? Use either dominos or a window with 2 panes to show your solutions.
3. I have two boxes and 15 marbles. How many marbles could be in each box?

If we think about all representations of number 5 and challenge our students in primary grades to see in how many ways they can have 5 marbles represented, we are guiding them to consider a variety of algebraic statements, for example 2+x=5, x+y=5

The windows concept was discussed as a result of one groups' use of "the magic square" idea in solving their problem. Working on either a concrete or symbolic level, they move their "marbles" around each pane of the window until they discover sets that solve each of the solutions of the problem. Using manipulatives, students role-played taking on the identity of people named on the cards. They shared marbles so that every person had a set that matched the numbers that fit into the equation on their individual cards.

What is significant about this? In kindergarten, students manipulate objects to solve problems. In middle and possibly high school settings, given a sheet of paper with the "windows table", students work out the manipulations on a more symbolic level. With older students who were introduced to algebraic concepts using manipulatives in the primary grades, many are able to work out the problems using a strategy that begins at the symbolic/abstract level. For students who have not had such experiences in the primary grades, teachers must use a strategy that begins at the pictorial/symbolic level, providing them with the scaffolding needed to attain the abstract level (solving a system of equations).

By demonstrating this sequence of activities (Marbles activity, sharing solutions, reflecting on the process) in a variety of representations, we were able to show how teachers could apply scaffolded differentiation using pedagogical content knowledge from their own classrooms. This provided additional evidence for them that students learn best when instruction is differentiated through a variety of representations.

Reflecting on Teaching in Context: Differentiation through Scaffolding

One of our goals has been to demonstrate for teachers how to use process and content standards in teaching mathematics. And at the same time, we modeled for them how to apply strategies for differentiation as a way of thinking versus as an add-on for one or a few particular students. In this attempt to teach pre-service and practicing teachers how to teach algebra, we found ourselves scaffolding our own instruction to support their learning. Teachers' understanding of algebra fell along a continuum of novice to expert. The process of differentiation was implemented and made explicit via conceptual and metacognitive scaffolding. Using a series of activities to facilitate the learning of algebra-related pedagogical content knowledge, we guided them to understanding through a series of specific open-ended questions. This captures what happens in effective teacher education, the dynamic intertwining of teaching content and teaching how to teach.

To check their understanding of differentiation, teachers were given the opportunity to differentiate the Marbles activity for students in their own K-8 classrooms. We found it very significant that when we asked them *if* they could adapt this algebraic activity to accommodate for learners with different abilities, they responded that it was not appropriate for younger or low functioning students. When we asked them to tell *how* they could adapt this activity to accommodate for learners with different abilities, some of them readily applied strategies for adapting instruction that were introduced earlier in the semester.

In teaching the Marbles activity over time, two instances occurred repeatedly: (a) groups stopped after finding one solution to the Marbles problem; and (b) teachers indicated that the Marbles problem could not be solved by students younger than middle school. Even though we planned to model differentiation all along, we did not expect the teachers to respond as described above and we were able to use these pivotal instances to reinforce the process for deciding when and how to differentiate.

Discussion: Implications for Instruction

What is it about this activity that we find valuable?

Equations with Marbles is a rich problem along more than one axis because it: (a) has more than one solution; (b) offers various possibilities for modifications across student's developmental levels; (c) provides for modifications within the same classroom for diverse learners; and (d) illustrates to teachers the value of investing more time in a single learning activity that addresses many outcomes in depth versus a cursory covering of the adopted curriculum.

Bridging the gap between arithmetic and algebra. There is a long way to go from playing with dominos to solving algebraic system of equations: from concrete, to pictorial, to symbolic, to formal and abstract; the representational stages of concept development. Demonstration of and experience with this continuum from concrete, hands-on play with a variable to solving algebraic system of equations builds teachers' understanding of the process. Teachers understand that by asking children developmentally appropriate open-ended questions, they are also laying the foundation for understanding algebraic equations later on in their students' school careers.

Something for every K-8 classroom. When the algebraic problems and concepts are broken down to their component parts, students are engaged with and interact with those parts, building a foundation for the symbolic, then abstract properties of algebra introduced later on. When teachers recognize that algebra can be broken down into its component parts, they will introduce it as such in an integrated fashion and reach each student at their current developmental level. When kids ask "why do we need to know this", teachers readily respond with real-life opportunities to use these skills (e.g. from organizing and classifying to transportation and scheduling problems).

What is it about this approach to teaching that we find valuable?

In teaching for understanding, we must differentiate instruction. The support for this point of view was magnified in repeated administrations of the Marbles activity when the following phenomena occurred over and over. We found it significant that teachers consistently (a) gave up after finding only one solution, and (b) believed the algebra concepts embedded in the Marbles activity could not be understood by students younger than middle school. It became imperative that we make it explicit to the teachers so that they too would not put unnecessary limits on their thinking and ultimately, their students'. Each time the event repeated itself, we prompted them to think beyond their current level of background experiences and beliefs to "try another way", to engage their pedagogical content understanding to reach all students. These instances provided the venue for the conceptual and metacognitive scaffolding processes as we prompted the teachers to differentiate. Experiencing these processes became the stepping stones in the teachers' learning to teach for understanding.

Critical thinking skills: Learning to think, to reason, to problem solve in creative ways

The above activities also provided opportunities for teachers to discuss and practice the development of the thinking skills that relate to applying relevant mathematics knowledge, skills and understandings. Among critical thinking skills are the abilities to

- ask relevant questions, create and describe mathematical problems or inquires, plan how to solve them, and explore *What if?* questions (*Inquiry skills*)
- recognize/collect relevant data and sort, organize, classify and interpret the information (*Information-processing skills*)

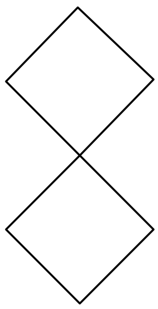
- identify patterns, generalize, give reasons for answers and conclusions, defend their solutions (*Reasoning skills)*
- apply acquired mathematical skills, broaden and connect ideas both within mathematics and with other areas, communicate their ideas to others, solve problems in innovative ways, and ask and investigate new questions (*Creative-thinking skills*)
- evaluate information, assess progress as they work in mathematics, verify solutions and suggestions critically, defend their methods, and justify choices of communication of their strategies/procedures and results (A*ssessment skills*)

In an inquiry-oriented classroom, open-ended rich activities similar to *Marbles* provide teachers with opportunities to facilitate the development of these essential thinking skills that every student needs to develop. A variety of activities similar to *Marbles* can be generated by changing numbers of variables and number of equations, or simply creating a different story around the same algebraic problem to make it more real-life connected and therefore more meaningful for both teachers and students.

Another Example: Robotics Design Studio

There are many other rich, open-ended problems, both in terms of variety of strategies and number of solutions, addressing mathematics standards and providing for developing critical thinking skills (NCTM 2000). The following example is one that authors of this paper also utilize often in their classrooms.

How to program a Lego Mindstorm robot (http://mindstorms.lego.com/) or Logo Blocks (http://llk.media.mit.edu/projects/cricket/software/index.shtm) so that a figure "eight", as one bellow, can be traced?



This open ended problem integrates many skills; addresses number of mathematics standards and can be both (a) simplified into simple subtasks, and (b) extended in various directions. Possible prompts include: What "commands" are necessary for a robot to move forward for 2 seconds? What command will allow for a 90^0 angle? How about a square path? What next? … In addition, working on a triangle path will provide great opportunity to have a hands-on experience with internal and exterior angles of a triangle. To integrate algebra, include problems with measuring, mapping and graphing. Keeping records of unsuccessful attempts and decision-making deepen critical thinking skills.

Decision-making versus cook-book solutions

The richness of the Marbles activity from both an algebraic and pedagogical point of view provided optimal conditions for the demonstration and use of conceptual and metacognitive scaffolding tools. These tools are built-in by designing appropriate questions. Using effective questioning techniques, teachers are stimulated to think how to differentiate versus to only respond to the next standard in the curriculum guide or teaching manual. When teachers were given rich open-ended questions, it provided them with the opportunity to think, find different solutions and try other ways to get students engaged in learning at their individual developmental levels. From our experiences, it is not modeling effective teaching strategies alone, but the process of reflective practice modeled and applied that we anticipate will put the conceptual and metacognitive scaffolding tools into action in their own classrooms.

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Differentiated instruction is a way of modifying the learning experience to meet students where they are. Because no two students learn the same way, differentiating a lesson makes a teacherâ€™s curriculum more accessible to students whose abilities vary from their peers. It ensures that high-quality instruction reaches every student in the classroom. Who Benefits from Differentiated Instruction? The goal of differentiating a lesson is to provide universal access to instruction while maintaining consistent learning objectives across student groups. While differentiated instruction helps all stud Differentiating instruction means that you observe and understand the differences and similarities among students and use this information to plan instruction. Here is a list of some key principles that form the foundation of differentiating instruction. Ongoing, formative assessment: Teachers continually assess to identify studentsâ€™ strengths and areas of need so they can meet students where they are and help them move forward.Â This is a powerful statistic that we teachers need to remember and act upon as we teach reading. Right now, too many middle schools place students in a curriculum in which everyone reads the same text and completes the same assignments. Unfortunately, this leaves too many students behind instead of moving them forward (Tomlinson, 2002). What is differentiated instruction? What does it mean for the new 'digital classroom', how can instructors leverage insights, and build an engaged class?Â For instructors, differentiating instruction according to individual student needs is the key to success in the virtual classroom. Top Hat Staff. â– May 26, 2020. In adapting to the â€˜new normalâ€™ of digital classrooms during COVID-19, the initial focus was simply moving the curriculum entirely online. But now educators are looking at ways to adapt their lesson plans to this new paradigm to make class time more engagingâ€”and more effective. Educators are well aware that students learn in different ways and at different paces.