Physics 514: Mechanics, Fluids, Chaos
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"There is a story about Heisenberg on his deathbed, declaring that he will have two questions for God: Why relativity? and Why turbulence? Heisenberg says, 'I really think He may have an answer to the first question.'" — Gleick, *Chaos*

I'll cover the mechanics of Lagrange and Hamilton, then move to the basics of fluid mechanics, a classical field of ever-growing importance to soft condensed matter physics and applied physics. Finally I'll discuss the amazing way in which simple systems, with few degrees of freedom and obeying simple deterministic equations of motion, can behave erratically. Such "chaotic" systems are all around us; recent improvements in our understanding of such processes have had a revolutionary effect on our modern conception of classical mechanics. I'll also focus on the spontaneous formation of structure in driven dissipative systems, a phenomenon which is fast becoming a paradigm for biology as well as physics. I also want the course to serve as an introduction to the qualitative theory of nonlinear differential equations, a subject of great use outside of physics.

General prerequisites:

Intermediate mechanics. Differential equations.

Books:

Buy these:

- A. Fetter, D. Walecka, *Theoretical mechanics of particles and continua.*
- E. Ott, *Chaos in dynamical systems* (Cambridge University Press, 1993).

I also think you'll enjoy the books in the "popular" section below. Don't be ashamed — I read them all. They're excellent.

Other books you will find helpful:

Mechanics:

- J. José and E. Saletan, *Classical dynamics: a contemporary approach* (CUP, 1999).
- L. Landau and E. Lifshitz, *Mechanics*

Fluids:

- Acheson, *Elementary fluid dynamics* (Oxford).
- G.K. Batchelor, *An introduction to fluid dynamics* (Cambridge University Press, 1967).
- J.P. Boon and S. Yip, *Molecular hydrodynamics* (Dover, 1980).
- P. Chaikin and T. Lubensky, *Principles of condensed matter physics.*
- S. Chandrasekhar, *Hydrodynamic and hydromagnetic stability* (Dover, 1961).
- T. Faber, *Fluid dynamics for physicists (Cambridge).*
- K. Huang, *Statistical mechanics*, 2^{nd} ed. (Wiley, 1987).
- L.D. Landau and E.M. Lifshitz, *Fluid mechanics*, 2^{nd} ed. (Pergamon, 1987).
- Levich, *Physico-chemical hydrodynamics* (Prentics 1962).
- J. Lighthill, An informal introduction to fluid mechanics (Oxford, 1989)
- B. Shivamoggi, *Theoretical Fluid Mechanics* (Wiley, 1998).
- D.J. Tritton, *Physical fluid dynamics* (Van Nostrand Reinhold, 1977).
- J. Welty, C. Wicks, R. Wilson, *Fundamentals of momentum, heat, and mass transfer* (Wiley, 1984).

Nonlinear Dynamics:

• H. Abarbanel, M.I. Rabinovich, M.M. Sushchik, *Introduction to nonlinear dynamics for physicists* (World Scientific, 1993).
• K. Alligood, T. D. Sauer, J. A. Yorke *Chaos: an introduction to dynamical systems* (Springer, 1997).
• Bergé, Pomeau, Vidal, *Order within chaos* (Wiley).
• P. Drazin, *Nonlinear systems* (CUP).
• M. Gutzwiller, *Chaos in classical and quantum mechanics*
• H. Haken, *Synergetics* (Springer, 1983).
• N. Hall, *Exploring Chaos* (introductory review articles)
• Hao Bai-lin, *Chaos II*.
• R Hilborn, *Chaos and nonlinear dynamics* (Oxford, 1994)
• P. Manneville, *Dissipative structures and weak turbulence* (Academic Press, 1990).
• E. Ott, *Chaos in dynamical systems* (Cambridge University Press, 1993).
• H. Schuster, *Deterministic Chaos*, 2nd ed.
• S. Strogatz, *Nonlinear dynamics and chaos with applications in physics, biology, chemistry, and engineering* (Addison-Wesley, 1994).
• Tabor, *Chaos and integrability in nonlinear dynamical systems* (Wiley, 1989).

Biophysical:

• H. Berg, *Random walks in biology*.
• Glass and Mackey, *From clocks to chaos* (Princeton, 1988).
• L. Glass et al, *Theory of Heart* (Springer, 1991).
• Goldbeter, *Biochemical Oscillations*.
• S. Kauffman, *The origins of order*.
• J. Lighthill, *Mathematical biofluidmechanics* (SIAM, 1975).
• J. D. Murray, *Mathematical biology* 2nd ed (Springer, 1993).
• S. Vogel, *Life in moving fluids: the physical biology of flow* 2^{nd} ed. (Princeton University Press, 1994).
• A. T. Winfree, *The geometry of biological time*.

Computational:

• G. Baker and J. Gollub, *Chaotic dynamics*.
• R. Enns and G. McGuire, *Nonlinear physics with maple for scientists and engineers*.
• R. Greene, *Classical mechanics with maple* (Springer, 1995).
• Hannon, *Dynamic modeling* (Springer).
• Hubbard, *Macmath: a dynamical systems software package for the Macintosh*, 2nd ed (Springer)
• Nusse and Yorke, *Dynamics*

Mathematics:

• R. Aris, *Vectors, tensors, and the basic equations of fluid mechanics* (Dover).
• V. Arnold, *Mathematical methods of classical mechanics*, 2d edition (Springer)
• G. I. Barenblatt, *Scaling, self-similarity, and intermediate asymptotics* (CUP, 1996).
• A. Das, *Integrable models*.
• P.G. Drazin, R.S. Johnson, *Solitons: an introduction* (Cambridge University Press, 1989).
• J. Guckenheimer and P. Holmes, *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields*.
• S. Lichtenberg, M. Lieberman, *Regular and Chaotic motion* 2nd ed. (Springer).
• L. Segel, *Mathematics applied to continuum mechanics* (Dover, 1987); C.C. Lin and L. Segel, *Mathematics applied to deterministic problems in the natural sciences,* (Macmillan, 1974); L. Segel, ed., *Mathematical models in molecular and cellular biology* (Cambridge University Press, 1980).
• L. Sparrow, *The Lorenz equations* (Springer 1982).
• M. Toda, *Nonlinear waves and solitons* (Kluwer, 1983).
• N. Woodhouse, *Geometric quantization*.

Popular/Historical:

• V. Arnold, *Huygens and Barrow, Newton and Hooke.*
• R. Feynman, *The character of physical law.*
• S. Gindikin, *Tales of mathematicians and physicists* (Birkhäuser).
• J. Gleick, *Chaos.*
• E. Lorenz, *The essence of chaos* (U. of Washington Press, 1993).
• D. Ruelle, *Chance and chaos* (Princeton University Press, 1991).
• I. Stewart, *Does God Play Dice? The new mathematics of chaos* (Penguin, 1990).
• M. Waldrop, *Complexity.*
• R. Westfall, *Never at rest: a biography of Isaac Newton* (Cambridge, 1980).

Miscellaneous:

• P. Anderson et al, eds, *The economy as an evolving complex system* (Addison-Wesley, 1988).
• V. Arnold, *Catastrophe theory* 3d ed.
• J. Binney and S. Tremaine, *Galactic dynamics* (Princeton University Press, 1987)
• R.J. Creswick, H.A. Farach, C.P. Poole, Jr., *Introduction to renormalization group methods in physics* (Wiley, 1992).
• L. Landau and E. Lifshitz, *Theory of elasticity*
• Ott, Sauer, and Yorke, eds., *Coping with chaos.*
• Frank H. Shu, *The physics of astrophysics* vols. 1–2 (University Science Books, 1991-1992).
• van Dyke, *An Album of Fluid Motion.*
• C.O. Weiss, R. Vilaseca, *Dynamics of lasers* (VCH, 1991).

Outline

[Optional sections in brackets]

Prologue: 17th Century Physics*

"It may be that the universal history is the history of the different intonations given a handful of metaphors." — Borges

1. Bird's eye view of this course. What is physics, anyway?
2. Two case studies. Introduction to basic nonlinear phenomena. Hopf and pitchfork bifurcations.
3. Historical essay. Newton: Precise measurement supersedes philosophical mumbo-jumbo. Newton's cat.

Part one: 18th Century Physics

"Nature, like a cautious testator, ties up her estate so as not to bestow it all on one generation, but has equal regard to the next, and the next, and the fortieth age." — R. W. Emerson

4. Lagrange: invariant form of Newton's law. Systems with constraints.
5. Hamilton: least action. Variational calculus.
6. Noether: symmetry and conservation laws. Systems of changing scope; locomotion of squids.
7. Small vibrations, normal modes. The role of symmetry. Debye theory of specific heat; Brillouin zone.
8. Damped and driven linear oscillations. The highly damped case; singular perturbations and boundary layers.
9. [Parametric resonance.] Nonlinear vibrations. Mass renormalization foreshadowed.
10. [Rigid bodies. Nutation. Tops and motion on groups.]

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* **Don't take these centuries too literally.**

11. Hamilton: symmetrical formulation. Examples. Liouville's theorem. Boltzmann: the bridge to statistical mechanics.
12. Poisson: the bracket formalism. Gorgeous form of Noether's theorem. Dirac: the bridge to quantum mechanics. Canonical transformations. A side trip to symplectic space. [Schrödinger's road to quantum mechanics. Hamilton-Jacobi theory. Kolmogoroff: when will perturbations destroy quasiperiodic behavior?]

12A. [Constrained Hamiltonian Dynamics. Dirac brackets.]

Part two: 19^{th} Century Physics

"It behoves us to remember that in physics it has taken great scientists to discover simple things. They are very great names indeed which we couple with the explanation of the path of a stone, the droop of a chain, the tints of a bubble, the shadows in a cup." — D'Arcy Thompson, 1917

13. What is a fluid? Viscosity versus shear modulus; the ice-cream machine test. The Newtonian idealization. Viscoelasticity. The critical force; what it's like being a bacterium. Microscopic origin of viscous drag; the link to entropy. Einstein–Smoluchowski: fluctuation–dissipation foreshadowed.
14. Flow. Blood circulation. Streamlines as useful lies. Quicksand; an unfriendly regatta. The conservation law.
15. Ideal fluids. Bernoulli equation. Hydrostatic equilibrium. Manometry; lift on wings; d'Alembert's paradox. Euler and Navier-Stokes for slobs. Phenomenology: physicists *are* basically slobs. The Most Important Idea in Physics.
16. Stress. Transport of momentum. Poiseuille flow and how to pronounce it. Why red blood cells are a tight squeeze.
17. A brush with thermodynamics. The complete equations of Newtonian fluids. The Prandtl number.
18. Reynolds number; our first dynamical scaling principle. Why wind tunnels work. Torrents versus tempests. Stokes' formula; sedimentation of macromolecules in an ultracentrifuge.
19. Vorticity. Irrotational flow. Magnus force. Kelvin's theorem. Vortex dynamics; smoke rings.
20. Ideal fluid paradoxes resolved: boundary conditions and boundary layers. How and when to simplify the complete equations. Separation; icy wings and plane crashes.
21. [Strong turbulence. Kolmogoroff scaling.]

22A. [Nonlinear waves. Hodgkin-Huxley theory of neuron action potentials.]

22B. [Solitary waves; solitons; Lax's amazing discovery.]

22C. [Shock waves; Burgers equation.]

23. Nonlinear pattern formation: Bénard instability; Boussinesq approximation. The Lorenz equations.

23A. [Dissipative structure.]

23B. [Stability of fluid disks: galactic structure.]

Part three: 20^{th} Century Physics

"To an anthropologist, the social reception of invention reminds one of the manner in which a strange young male is first repulsed, then tolerated, upon the fringes of a group of howler monkeys he wishes to join. Finally, since the memories of the animals are short, he becomes familiar, is accepted, and fades into the mass. In a similar way the discoveries of Darwin and Wallace were at first castigated and then by degrees absorbed." — Loren Eiseley

24. Bifurcations revisited. Phenomenology of Lorenz equations. Lyapunov exponents. Chaos defined. Discovery of a hidden iterated map.
25. Survey of chaos, basic phenomena. Driven pendulum, Bénard convection, Belusov-Zhabotinsky system, quasiperiodic systems. Diagnostics of chaos. Simplest iterated map: the binary shift. The Incredibly Simple Secret of Chaos.
26. Iterated maps. The logistic map: sex+violence→chaos. Doubling bifurcation. Feigenbaum universality. Periodic windows and the intermittency route to chaos.
27. Fractals.
28. Strange attractors. Cowboys versus scholars. Attractor reconstruction from experimental data.
29. Quasiperiodicity, mode locking. Chirikov standard map, destruction of toroidal orbits, comparison to experiments.

Epilogue: 21st Century Physics?

"Les Philosophes qui font des systèmes sur la secrète construction de l'univers, sont comme nos voyageurs qui vont à Constantinople, et qui parlent du Sérail: Ils n'en ont vu que le dehors, et ils prétendent savoir ce que fait le Sultan avec ses Favorites." — Voltaire

30. Biorhythms. Dynamics of cardiac tissue. Control of chaos.
31. Valedictory. Would Newton be pleased?

Literature References:

H. Abarbanel et al, Rev. Mod. Phys. 65 (1993).

G. Ahlers, "Experiments on bifurcations and 1d patterns in nonlinear systems far from equilibrium," in E. Jen, ed., *Lectures in the sciences of complexity* (Addison-Wesley, 1989).

A. Andreev et al, "Quantum Chaos..." PRL 76 (1976).

B. Arthur, "Positive feedbacks in the economy," Scientific American 2/1990.

Y. Braiman et al, "Taming spatiotemporal chaos with disorder," Nature 378 (1995) 465–7.

D. R. Chialvo, R. F. Gilmour, and Jose Jalife, "Low dimensional chaos in cardiac tissue," Nature 343 (1990) 653–7.

P. E. Cladis and P. Palffy-Muhoray, eds., *Spatio-temporal patterns in nonequilibrium complex systems* (Addison-Wesley, 1995).

M.C. Cross and P.C. Hohenberg, "Pattern formation outside of equilibrium," *Reviews of Modern Physics* **65**, 851-1112 (1993); "An introduction to pattern formation in nonequilibrium systems," in *Fluctuations and stochastic processes in condensed matter,* ed. A. Garrido (Springer 1987).

M. Feigenbaum, in *Asymptotic realms of physics,* ed. A.H. Guth, K. Huang, and R.L. Jaffe (MIT, 1983).

N. Go et al, "Dynamics of a small globular protein in terms of low-frequency vibrational modes," PNAS 80 (1983)3696; B. Brooks and M. Karplus, "Harmonic dynamics of proteins: normal modes and fluctuations," PNAS 80 (1983) 6571.

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J. Glazier and A. Libchaber, "Quasiperiodicity and dynamical systems: an experimentalist's view," IEEE Trans Circuits Systems **35** (1988) 790–809.

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J. Langer, in *Chance and Matter* (North-Holland, 1987) [pattern formation lectures]; Langer JS Dendrites, viscous fingers, and the theory of pattern-formation Science 243 1150-1156 (1989); Gollub, J.P.; Langer, J.S. Pattern formation in nonequilibrium physics. Reviews of Modern Physics, Feb. 1999, vol.71 S396-403.

N. Lebovitz, ed *Fluid Dynamics in Astrophysics and Geophysics* (AMS 1983).

P. S. Linsay, "Period doubling and chaotic behavior in a driven anharmonic oscillator," PRL 47 (1981) 1349–52.

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A. Newell, "The dynamics and analysis of patterns," in E. Jen, ed., *Lectures in the sciences of complexity* (Addison-Wesley, 1989).

E. Ott, Grebogi, and J. Yorke, "Controlling chaos," PRL 64 (1990) 1196.

E. Purcell, "Life at low Reynolds number," Am J Phys 45 (77) 3.

Lord Rayleigh, "On the instability of jets," Proc. Lond. Math. Soc. **10** (1879) 4; "On the instability of a cylinder of viscous liquid under capillary force," Phil. Mag. **34** (1892) 145 [see his *Collected Works*].

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W. Yourgrau and S. Mandelstam, *Variational principles in dynamics and quantum theory* 3d ed. (Saunders 1968) [includes variational principles for hydrodynamics]

Physics of Fluids. Filter by Journal: All Journals AIP Advances APL Bioengineering APL Materials APL Photonics Applied Physics Letters Applied Physics Reviews AVS Quantum Science Biointerphases Biomicrofluidics Biophysics Reviews Chaos Chemical Physics Reviews Journal of Applied Physics Journal of Chemical Physics Journal of Laser Applications Journal of Mathematical Physics Journal of Physical and Chemical Reference Data Journal of Renewable and Sustainable Energy Journal of Vacuum Science & Technology A Journal. of Vacuum Science & Technology B Physics of Fluids Physics of Plasmas Review of Physics of Fluids (PoF) is a preeminent journal devoted to publishing original theoretical, computational, and experimental contributions to the understanding of the dynamics of gases, liquids, and complex or multiphase fluids. Topics published in PoF are diverse and reflect the most important subjects in fluid dynamics, including, but not limited to: -Acoustics -Aerospace and aeronautical flow -Astrophysical flow -Biofluid mechanics -Cavitation and cavitating flows -Combustion flows -Complex fluids -Compressible flow -Computational fluid dynamics -Contact lines -Continuum mechanics -Convectio FLUID MECHANICS CONCEPT The term "fluid" in everyday language typically refers only to liquids, but in the realm of physics, fluid describes any gas or liquid that conforms to the shape of its container. Fluid mechanics is the study of gases and liquids at rest and in motion.Â This area of physics is divided into fluid statics, the study of the behavior of stationary fluids, and fluid dynamics, the study of the behavior of moving, or flowing, fluids. Fluid dynamics is further divided into hydrodynamics, or the study of water flow, and aerodynamics, the study of airflow. Applications of fluid mechanics include a variety of machines, ranging from the water-wheel to the airplane.