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Sommersemester 2007

Seminarankündigung SS 2007

Seminar zur Globalen Analysis B

Cyclic cohomology and the noncommutative Chern character

Noncommutative Geometry is an area of mathematics which has been dominated by the work of Alain Connes in the last 20-25 years. The basic idea is that instead of point sets (e.g. manifolds) one studies the coordinate ring of (smooth) functions. This point of view has been around in algebraic geometry for decades but it was Connes' who showed that also manifolds and index theory can be understood from this perspective. Examples of 'noncommutative spaces', where now the coordinate ring is a noncommutative algebra, are abundant and Noncommutative Geometry is an area of active current research.

The purpose of this seminar is modest. We want to study some of the basic material of noncommutative geometry, like cyclic (co)homology, Fredholm modules, the noncommutative analogue of the classical Chern character and the Hochschild–Kostant–Rosenberg–Connes Theorem.

For a first reading we will use the excellent survey by Higson [**11**].

Since I also hope to attract international graduate students the seminar is going to be held in English.

When and Where: Flexible. Options: Do, 8-10, SR B **or** Mo, 14-16, SR A

First meeting: Mo, 02.04.2007, 14:15, Beringstr. 4, Seminarraum A.

If you are interested you should informally contact Matthias Lesch (lesch@math.uni-bonn.de) or Michael Bohn (mbohn@math.uni-bonn.de)

Talks

1. The Gelfand–Naimark Theorem and noncommutative topology.

Material: [**2**], in particular the introductory section on (commutative) K–theory, [**15**, Sec. 4.2], [**10**, Chap. I]

Explain the content of the Gelfand–Naimark theorem and the basic dictionary of non–commutative topology. This dictionary expresses basic topological notions in terms of the algebra of continuous functions. Review the main results of K–theory for C^*–algebras.

2. The trace and the Schatten ideals.

Material: [**15**, Sec. 3.4], [**16**]

Follow the exposition in [**15**, Sec. 3.4] and introduce the trace and trace class operators on a Hilbert space. The trace is a positive linear functional, taking possibly the value $+\infty$, on the cone of nonnegative operators. The theory of the trace in many respects resembles measure theory. This should be emphasized and therefore also the so–called Schatten p–ideals [**15**, E. 3.4.2-E.3.4.4] should be discussed. Another crucial result is the fact that the map $(S,T) \mapsto \operatorname{tr}(ST)$, where S is bounded and T is trace class, implements the duality between the Banach space of trace class operators and the Banach space of bounded linear operators. This generalizes the well–known duality between L^1 and L^∞ in measure theory.

In addition to [**15**] one should briefly address the problem whether for a trace class operator T the sequence of eigenvalues is summable and $\operatorname{tr}(T)$ equals the sum of the eigenvalues. For self–adjoint operators this is easy to see and it should be presented. The general case, the so–called Lidskii–Theorem, is more difficult. Consult [**16**] and give at least a couple of comments.

3. Fredholm modules and the index pairing.

Material: [**11**, Sec. 2.1], [**3**, Appendix], [**4**, Appendix IV.A], [**12**, Chap. 8]

The material of [**11**, 2.1] should be covered. However, some more background on the role of Fredholm modules as cycles in K–homology should be given. K–homology is the dual theory to K–theory and there is a natural bilinear pairing

$$\operatorname{index} : K_j(A) \times K^j(A) \longrightarrow \mathbb{Z}.$$

Since Fredholm modules are the cycles in K–homology, every Fredholm module induces naturally a map $K_j(A) \to \mathbb{C}$. This map should be explained in detail in both the even and in the odd case ([**11**, Sec. 2.1], [**4**, Prop. IV.2]).

Explain the difference between algebraic K_j and topological K_j for $j = 0, 1$.

Then *finitely summable* Fredholm modules should be discussed. The following simple but important Lemma should be proved:

Proposition. *Let H be a Hilbert space and $T \in \mathcal{B}(H)$ a Fredholm operator. Furthermore let $S \in \mathcal{B}(H)$ be a parametrix of T such that $(I - ST)^k, (I - TS)^k$ are trace class for some integer $k \geq 1$. Then*

$$\operatorname{ind}(T) = \operatorname{tr}((I - ST)^k) - \operatorname{tr}((I - TS)^k).$$

4. The character of a finitely summable Fredholm module.

Material: [**11**, Sec. 2.2], [**4**, Sec. IV.1],

Present [**11**, Sec. 2.2] giving full proofs of Theorems 2.10 and Theorem 2.11. For more details consult [**4**, Sec. IV.1]. Try to give a proof of Theorem 2.7 without becoming too technical about cyclic cohomology. Consult [**3**].

5. Hochschild (co)homology.

Material: [**14**, Chap I], [**11**, Sec. 2.3], [**7**], [**18**]

Concentrate on Sections 1.0, 1.1, 1.5 and 1.6 of Loday's book. You may use the framework of simplicial modules but you should avoid a too high level of abstraction. The main examples of simplicial modules are the Hochschild complex and co-complex.

The bar resolution Prop. 1.1.12 and the normalized Hochschild complex (1.1.14) should be discussion thoroughly. Loday uses a spectral sequence argument. Also for later purposes it might be helpful to give a concise presentation of the spectral sequence of a double complex [**14**, Appendix D].

It will help to demystify the spectral sequence argument if you give a short direct proof of the acyclicity of the normalized Hochschild complex using the displayed formula in the proof of [**14**, Lemma 1.6.6].

6./7. Cyclic (co)homology I and II.

Material: [**14**, Chap II], [**11**, Sec. 2.3], [**4**, Sec. III.1, III.3], [**7**], [**18**]

Follow Loday's book and present the various complexes which calculate cyclic (co)homology (cyclic bicomplex, Connes' complex, Connes' bB–bicomplex). Connes' SBI–sequence and the periodicity operator as well as period cyclic (co)homology should be covered, too. Note that in Loday's book the periodic theory is discussed much later in Chap. V.

Another topic which should be discussed is the notion of the character of a cycle [**4**, Sec. III.1.α]. This gives a natural way to construct cyclic cocycles. This sheds new light on the 4. talk. Finally present Theorems 2.19 and 2.21 in [**11**, Sec. 2.3].

8. The Hochschild–Kostant–Rosenberg–Connes Theorem.

Material: [**11**, Sec. 2.4], [**3**, Theorem 46], [**10**, Sec. 8.5]

For the algebra of smooth functions on a manifold the (continuous) Hochschild homology is canonically isomorphic to the space of differential forms and the (continuous) periodic cyclic homology is canonically isomorphic to the $\mathbb{Z}_2$–graded de Rham cohomology. This is the content of the so–called HKRC–Theorem. As a consequence Hochschild homology may be viewed as the noncommutative substitute to differential forms and periodic cyclic homology may be viewed as the noncommutative substitute for de Rham cohomology.

A proof of this important theorem can be found in [**3**] and in [**10**]. There exists a relatively new proof due to Teleman (see the references in [**10**]).

9./10. The noncommutative Chern character I and II.

Material: [**11**, Sec. 2.5], [**8**], [**9**]

For compact manifolds the Chern character is a natural transformation from K–cohomology to de Rham cohomology. In light of the HKRC–Theorem we should expect that the noncommutative Chern character is a natural transformation from K–theory to cyclic homology. This is indeed the case. Your presentation should follow the original papers by Getzler and Getzler–Szenes.

11. Further topics. There are various options for further topics (if time permits, the SS is short). One could e.g. discuss the Dixmier trace and the Hochschild character theorem [**11**, Sec. 3] and the Local Index Theorem of Connes and Moscovici [**6**].

References

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@inproceedings{Fuchs2006CyclicCA, title={Cyclic Cohomology and Higher Rank Lattices}, author={Mathias Fuchs}, year={2006} }. Mathias Fuchs. Published 2006. Mathematics. We give a new proof of the absence of non-trivial idempotents in the group ring of torsion-free cocompact lattices in SL(n,C). It is based on the following procedure.Â We then perform a Dirac-dual Dirac method on smooth algebras in analytic cyclic cohomology. This is based on a form of equivariant Bott periodicity underâ€¦Â Local Cyclic Cohomology of Group Banach Algebras and the bivariant Chern-Connes Character of the Î³-element. Michael Puschnigg. 2007. Highly Influential. View 4 excerpts, references background and methods. The Noncommutative Chern-Connes Character of the Locally Compact Quantum Normalizer of SU(1,1) in SL(2,C). October 2003. International Journal of Mathematics 15(4).Â We then use the technique of reduction to the maximal subgroup to compute the K-theory, the periodic cyclic homology and the corresponding Chem-Connes character. Discover the world's research. 19+ million members. We analyze the structure of the dual Chernâ€“Connes character from (analytic) K-homology to local cyclic cohomology under some reasonable hypotheses. We also investigate the twisted periodic cyclic homology via locally convex algebras and the local cyclic homology via. Câˆ—. -algebras (in the compact case). Article information. Source Kyoto J. Math., Volume 54, Number 3 (2014), 597-640. Dates First available in Project Euclid: 14 August 2014. Permanent link to this document https://projecteuclid.org/euclid.kjm/1408020880. 3.Connes, A.. Entire cyclic cohomology of Banach algebras and characters of Î¸-summable Fredholm modules. K-Theory 1(6):519â€“548, 1988.CrossRef Google Scholar. 4.Connes, A. and Cuntz, J.. Quasi homomorphismes, cohomologie cyclique et positivitÃ©.Â The residue index theorem of Connes and Moscovici. In Surveys in noncommutative geometry, Clay Math. Proc. 6, pages 71â€“126.Â Equivariant local cyclic homology and the equivariant Chern-Connes character. Doc. Math. 12:313â€“359 (electronic), 2007.Google Scholar. 31.Voigt, Christian. Equivariant periodic cyclic homology. J. Inst. Math. Jussieu 6(4):689â€“763, 2007.CrossRef Google Scholar. 32.Wodzicki, Mariusz. Excision in cyclic homology and in rational algebraic K-theory. The current study aimed to explore the linguistic analysis of neologism related to Coronavirus (COVID-19). Recently, a new coronavirus disease COVID-19 has emerged as a respiratory infection with significant concern for global public health hazards. However, with each passing day, more and more confirmed cases are being reported worldwide which has alarmed the global authorities including the World Health Organization (WHO). In this study, the researcher uses the term neologism which means the coinage of new words. Neologism played a significant role throughout the history of epidemic and pand